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Quantum critical Hall exponents

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ABSTRACT

We investigate a finite size "double scaling" hypothesis using data from an experiment on a quantum Hall system with short range disorder [1–3]. For Hall bars of width w at temperature T the scaling form is $w^{-\mu}T^{-\kappa}$, where the critical exponent $\mu \approx 0.23$ we extract from the data is comparable to the multi-fractal exponent $\alpha_0 - 2$ obtained from the Chalker–Coddington (CC) model [4]. We also use the data to find the approximate location (in the resistivity plane) of seven quantum critical points, all of which closely agree with the predictions derived long ago from the modular symmetry of a toroidal σ-model with m matter fields [5]. The value $\nu_8 = 2.60513\ldots$ of the localisation exponent obtained from the $m = 8$ model is in excellent agreement with the best available numerical value $\nu_{\text{num}} = 2.607 \pm 0.004$ derived from the CC-model [6]. Existing experimental data appear to favour the $m = 9$ model, suggesting that the quantum Hall system is not in the same universality class as the CC-model. We discuss the reason this may not be the case, and propose experimental tests to distinguish between the two possibilities.

One of the unsolved problems in the quantum Hall effect is to understand the critical behaviour of the delocalisation transition between Hall plateaux. This includes finding the position of quantum critical points ($\otimes$) in the resistivity plane (the upper half plane spanned by the Hall resistivity ρ_H and the direct resistivity $\rho_D \geqslant 0$), as well as determining the critical exponents characterising the universality class to which the system belongs. Progress on this problem has been slow, in part because of the paucity of experimental data. We are only aware of two experiments that can directly measure the correlation (localisation) length exponent ν [1,7]. It has also emerged that numerical simulations are sensitive to sub-leading corrections that were neglected in earlier work, leading to a substantial change in the value of ν_{num}.

As the temperature T drops into the quantum regime, temperature scaling has been observed over two decades in some quantum Hall devices, but this must eventually stop at a finite temperature $T_s > 0$, because the inelastic scattering length depends on temperature. When the sample is cold enough this length exceeds the sample size w, temperature scaling "saturates", and the response functions become independent of T (compare Fig. 1). The first experiment to probe this scenario measured the maximum slope of the Hall resistivity $\rho'_H = \max(d\rho_H/dB)$, and also the width $\Delta B_D \sim 1/\rho'_H$ between inflection points of ρ_D, in a GaAs–AlGaAs heterostructure etched with several Hall bars of different sizes w [7]. Above the saturation point ($T > T_s$) they found that $\rho'_H = uT^{-\kappa}$, and in this material the unsaturated data collapsed to a single line, corresponding to a w-independent prefactor u, similar to our Fig. 1(b). However, the slope κ of this line is not universal, as it depends on the amount and type of disorder, the transition and other details. A scaling argument [7] suggests that the saturation value ρ'_{Hs} is directly related to ν by $\rho'_{Hs} \sim w^{1/\nu}$. This exponent can also be obtained indirectly from the scaling relation $\kappa\nu z = 1$ by using $T_s \sim w^{-z}$ to find z, but if possible ν should be found directly from the data since it is difficult to obtain precise values of T_s. Using the direct method a remarkable degree of universality was found in [7], with $\nu = 2.3 \pm 0.1$, for different samples and for a number of distinct Hall transitions.

Subsequent work [1–3] has shown that the type of disorder determines if the temperature scaling exponent κ is universal. Fig. 1(a) shows our reconstruction of data obtained from an AlGaAs–AlGaAs heterostructure suitably doped so that the disorder potential is dominated by short range fluctuations [1,2]. In this case a reasonably universal value $\kappa = 0.42 \pm 0.01$ is obtained.

However, this universality appears to come at a price. The high-temperature data no longer collapse, because the prefactor u depends on the geometry of the Hall bar through w, and consequently ν cannot be read off directly from the saturation data. The value $\nu = 2.38$ (no error bars) given in [1] was obtained indirectly by using the scaling relation $\nu = 1/\kappa z$, with $\kappa = 0.42$ and $z = 1$, neither of which we find (below) to be the best fit to the full set of data.

We propose a way to recover the direct determination of ν, which follows from the observation that $\rho'_{Hs} \sim w^{\tau}$, where

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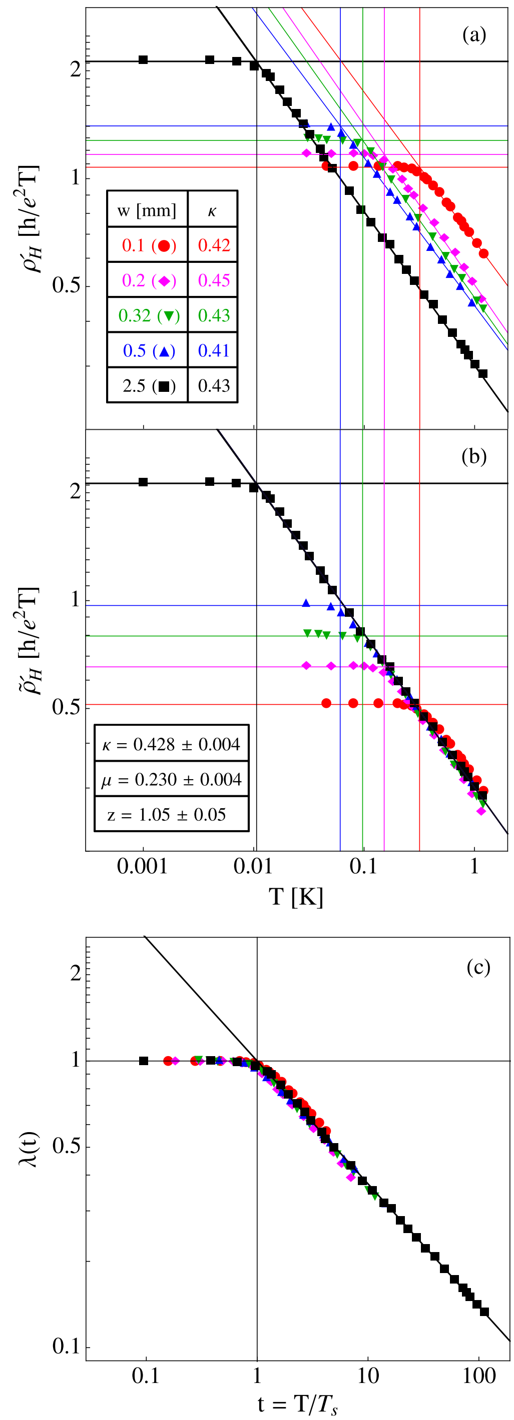

Fig. 1. (Colour online.) (a) Reconstruction of data for the maximum slope of the Hall resistivity $\rho'_H(T)$ from [1] ($w = 0.1, 0.5, 2.5$ mm) and [2] ($w = 0.2, 0.32$ mm). Horizontal and diagonal lines are fits to the data, whose intersection defines the saturation temperature T_s (vertical lines). Above saturation ρ'_H scales with exponent κ given in the inset table. Below saturation ρ'_H takes constant values ρ'_{Hs}. (b) Simultaneous fit of all data from (a) above saturation to the scaling form $\tilde{\rho}'_H$. (c) All data collapse to the dimensionless function λ if temperature is measured in T_s-units.

$\tau = 0.22 \pm 0.02$ is obtained directly from the data, as shown in Fig. 2(a). This can be recast as a finite size scaling hypothesis for the prefactor:

$$u(w) \sim w^{-\mu} \quad (\mu > 0), \tag{1}$$

where $\mu = \nu^{-1} - \tau$ is a new exponent, hopefully at least as universal as κ is. If this is true we should find that the unsaturated data

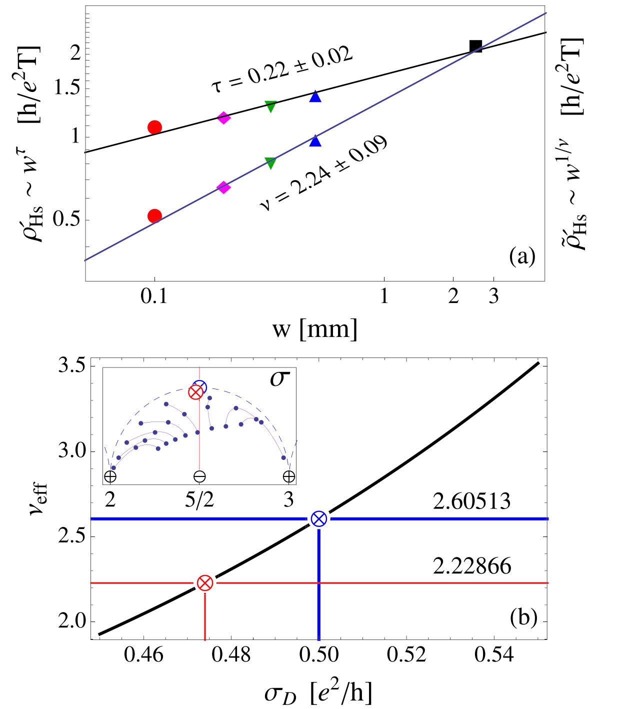

Fig. 2. (a) Best fit of the saturation data $\rho'_{Hs} \sim w^{\tau}$ from Fig. 1(a), and $\tilde{\rho}'_{Hs} \sim w^{1/\nu}$ from Fig. 1(b). (b) The "effective exponent" $\nu_{\mathrm{eff}}(\sigma_D)$ for the $m = 8$ model, with $\sigma_H = 5/2[\mathrm{e}^2/\mathrm{h}]$. Inset: the $2 \otimes 3$ transition in the σ-plane, with T-driven RG flow lines derived from data at $T = 31, 114, 510$ mK (blue dots) [3]. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

($T > T_s$) collapse to a single w-independent temperature scaling function:

$$\tilde{\rho}'_H = (w/w_{\max})^{\mu} \rho'_H \sim T^{-\kappa}, \tag{2}$$

for a single universal value of the exponent μ. We have chosen to hold the data for the largest Hall bar ($w_{\max} = 2.5$ mm) fixed, to serve as a reference. Fig. 1(b) shows the best global fit of all data [1,2] above saturation to this scaling form. The data below saturation do not collapse, and in fact move further apart. We can now extract ν *directly* from $\tilde{\rho}'_{Hs} \sim w^{1/\nu}$, compare Fig. 2(a), as was done in the first experiment [7]. We find that $\kappa = 0.428 \pm 0.004$ and $\mu = 0.230 \pm 0.004$ gives the best fit to the 71 unsaturated data points, with 95% confidence intervals (0.421, 0.436) and (0.222, 0.238). The double scaling property of ρ'_H allows us to collapse all data to the form $\lambda(t = T/T_s) \propto w^{\mu} T_s^{\kappa} \rho'_H$ shown in Fig. 1(c).

Our value for κ is higher, and the error is substantially smaller, than the value 0.42 ± 0.01 reported in [1], and remains essentially the same if we omit μ as a fitting parameter. The reason for this discrepancy is presumably that we are using the whole data set simultaneously. The average of the slopes computed independently for each data set ($w = 0.1, 0.2, 0.32, 0.5, 2.5$ mm) is $\bar{\kappa} = 0.427 \pm 0.011 \approx 0.43 \pm 0.01$, but the average of the slopes computed independently for the sub-set of data published in [1] ($w = 0.1, 0.5, 2.5$ mm) is $\bar{\kappa} = 0.419 \pm 0.007 \approx 0.42 \pm 0.01$. Our best fit to the scaling form $T_s \sim w^{-z}$ is $z = 1.05 \pm 0.05$, which gives (using our κ and $\kappa\nu z = 1$) $\nu = 2.23 \pm 0.11$, consistent with the value $\nu = 2.24 \pm 0.09$ obtained directly using the double scaling hypothesis (compare Fig. 2(a)).

The first experiment [7], on a sample with long range disorder potential where it was found that κ is not universal, has $\mu = 0$ because the data above saturation collapse. With only two experiments at our disposal we cannot conclude that μ is a universal exponent whose value can be used to label universality classes, but