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Condensational power of air circulation in the presence of a horizontal temperature gradient

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ABSTRACT

From the condition of hydrostatic equilibrium and energy conservation a general expression is derived for the power of air circulation induced by water vapor condensation in the presence of a horizontal gradient of temperature. It is shown that the obtained expression for circulation power agrees with the continuity equation. The impact of droplets that form upon condensation on the circulation power is evaluated. Theoretical estimates are compared with observational evidence.

1. Introduction

Phase transitions of water vapor in the air bring about pressure changes and lead to the appearance of pressure gradients that drive air circulation. Condensation of saturated water vapor occurs when the moist air ascends and, consequently, cools, as well as when moist air flows horizontally towards areas of lower temperature. The condition of hydrostatic equilibrium does not allow large vertical velocities to develop, such that all the condensational power released in the ascending air is translated to the power of horizontal winds [1–3].

Dissipation of the horizontal air flow due to friction on the Earth's surface leads to the formation of vortices. Turbulent diffusion enhances evaporation and mixing of water vapor in the air. Therefore, if the air moves towards an area where temperature and humidity are high enough to ensure sufficient evaporation, an increase in the partial pressure of water vapor by evaporation can significantly impede or even fully arrest the condensational air circulation.

In this work, we formulate a general equation for the power of condensational circulation of air in the case of arbitrary temperature gradients. We show that this equation is consistent with the continuity equation for air in the presence of the phase transitions of water vapor. It is also shown that, on average, formation of condensate particles reduces the power of condensational circulation by a small relative magnitude. A qualitative explanation of the observed peculiarities for global circulation of the atmosphere of Earth is given.

2. Dynamic equation of condensational air circulation

In the ascending air all its gaseous components, including water vapor, share the same vertical velocity **w**. In hydrostatic equilibrium, the increase of potential energy of the ascending air in unit volume is equal to the decrease of air pressure p, $(-\partial p/\partial z = \rho g)$, where ρ is air density, g is the acceleration of gravity, **w** and z-axis are directed opposite to **g**. This means that work is not produced on the ascending air, and the kinetic energy of air does not change. In the absence of condensation of water vapor, the relative partial pressures p_i/p of all air gases remain constant and independent of z, because the ascending gases share the same vertical velocity. It follows that partial pressures of the air gases, which all have different molar masses M_i, are equally distributed over height. Their distribution coincides with that of the air as a whole:

$$-\frac{1}{p_i}\frac{\partial p_i}{\partial z} = -\frac{1}{p}\frac{\partial p}{\partial z} = \frac{1}{h}, \quad h \equiv \frac{RT}{Mg}, \quad p = \rho g h, \tag{1}$$

where $M = \sum_i M_i p_i/p$ is molar mass of air, R is the universal gas constant, h is the scale height of the vertical distribution of all air gases (height of a uniformly dense air column). The last equality in (1) is equation of state for the ideal gas. The fulfillment of Eq. (1) for the ascending moist air with pressure p is referred to below as the condition of hydrostatic equilibrium.

In a stationary flow, when moist air ascends and cools, the water vapor condenses and leaves the gaseous phase above the horizontal surface (cloud base) where the water vapor is saturated. The vertical distribution of water vapor shrinks towards the Earth's surface and does not conform to Eq. (1):

$$-\frac{\partial p_v}{\partial z} = \frac{p_v}{h_c} \gg \frac{p_v}{h}, \qquad h^{-1} \ll h_c^{-1} = h_T^{-1}\xi,$$
$$h_T^{-1} \equiv -\frac{1}{T}\frac{\partial T}{\partial z}, \qquad \xi \equiv \frac{L_v}{RT}, \tag{2}$$

where h_c is determined by the Clausius–Clapeyron equation, L_v is the energy of vaporization (latent heat released during condensation) [1,3], $\xi \gg 1$ is dimensionless.

The fact that the vertical distributions (2) and (1) for the water vapor are different implies that when the ascent is accompanied by

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condensation, work is performed on the air that should generate kinetic energy with the following power [1–3]:

$$s = wp_v\left(\frac{1}{h_c} - \frac{1}{h}\right) = -w\left(\frac{\partial p_v}{\partial z} - \gamma\frac{\partial p}{\partial z}\right), \quad \gamma \equiv \frac{p_v}{p}. \tag{3}$$

Eq. (3) can be re-written in another convenient form:

$$s = wp_v\frac{1}{h_\gamma} = -wp\frac{\partial \gamma}{\partial z}, \quad -\frac{\partial \gamma}{\partial z} = \frac{\gamma}{h_\gamma}, \quad h_\gamma^{-1} \equiv h_c^{-1} - h^{-1}. \tag{4}$$

The power generated by water vapor with partial pressure p_v is of the order of $\gamma \ll 1$ as compared to the power generated by air as a whole with the total pressure p, for example, when the air is filling vacuum. By integrating power s (3) over time ($wdt = dz$), we find that total work corresponding to complete condensation of water vapor in the ascending air is of the order of $p_v = \gamma p$ where p_v and p are values at the Earth's surface ($z = 0$). This work is equal to the kinetic energy $\rho v^2/2 \sim \gamma p$ that would be acquired in the absence of friction by the air volume upon complete condensation of water vapor it contains ($\mathbf{v} = \mathbf{u} + \mathbf{w}$, where $\mathbf{u}$ is the horizontal velocity component). For $\gamma = 3 \times 10^{-2}$, $p = 10^5$ Pa and $\rho = 1.3\ \mathrm{kg\,m^{-3}}$ this corresponds to velocity $v = (2\gamma p/\rho)^{1/2} \approx 70\ \mathrm{m\,s^{-1}}$. Depending on the geometry of the circulation this kinetic energy produced by condensation corresponds to either vertical or horizontal motion or both.

In large condensation areas with horizontal size L significantly exceeding condensation height, the latter being of the order of the atmospheric scale height h, the observed annual mean vertical velocities of the ascending air are of the order of $1\ \mathrm{mm\,s^{-1}}$ [4]. This indicates that any significant vertical acceleration of the air is absent and that condition (1) of the hydrostatic equilibrium is preserved for the air as a whole with total pressure p. In this case, energy conservation and the condition of hydrostatic equilibrium dictate that power s (3) corresponds to the power of the horizontal air flow $-\mathbf{u}\nabla p$, where $\mathbf{u}$ is the vector of horizontal velocity [2]. Flow stationarity for circulating air corresponds to the equality between the characteristic times of the horizontal and vertical air motions (see Sections 3 and 4 below). Thus, the equality between the power released during the vertical motion of the ascending air (3) and the power of the horizontal air motion is equivalent to the equality of the corresponding amounts of work (energy conservation) and takes the form of the following dynamic relationship:

$$s = -\mathbf{w}(\nabla p_v - \gamma\nabla p) \equiv -p\mathbf{w}\nabla\gamma = -\mathbf{u}\nabla p. \tag{5}$$

Equality (5) holds true, if the horizontal air motion is not accompanied by phase transitions of water vapor for constant z, i.e. when $\mathbf{u}\nabla p_v = 0$ and $\mathbf{u}\nabla T = 0$ and the condition of a horizontal isothermality is fulfilled [2,3]. In this case, the surfaces of constant relative humidity (in particular, the cloud base surface where relative humidity is unity) remain horizontal in the entire circulation area.

It is however not difficult to generalize (5) to the case of a non-zero horizontal temperature gradient. In this case, to the left-hand side of the equality (5) one should add the power related to the change of water vapor partial pressure caused by evaporation or condensation in the horizontal air flow. This power is equal to $-\mathbf{u}\nabla p_v$. The general form of the dynamic equation for the power of condensational circulation (it was obtained from different physical considerations in [6]) is then given by the following relationship:

$$s = -p\mathbf{w}\nabla\gamma - \mathbf{u}\nabla p_v = -\mathbf{u}\nabla p. \tag{6}$$

A horizontal gradient of temperature can enhance condensation, if $-\mathbf{u}\nabla p_v > 0$, or weaken it by evaporation, if $-\mathbf{u}\nabla p_v < 0$ [6].

In coordinate axes x and z, where x is directed along $\mathbf{u}$, the horizontal pressure gradient in (6) has the form (see (1), (2) and (4) in [6]):

$$-\frac{\partial p}{\partial x} = \frac{p_v}{h_\gamma}\frac{w}{u}(1 - A), \qquad A \equiv -\frac{h_\gamma}{h_c}\frac{u}{w}\frac{\partial T/\partial x}{\partial T/\partial z} = \xi\frac{u}{w}\frac{h_\gamma}{T}\frac{\partial T}{\partial x}. \tag{7}$$

All variables in (7) can be retrieved from observations. As shown in [5] with the empirically measured values entered into the right-hand side of (7), this equation yields an estimate of the mean horizontal pressure gradient in the Amazon river basin that agrees well with observations.

3. Continuity equation in the presence of phase transitions of water vapor

In the stationary case, the continuity equations for the water vapor and the dry air constituents have the form

$$\nabla\mathbf{v}N_v \equiv N_v\nabla\mathbf{v} + \mathbf{v}\nabla N_v = -S, \quad \mathbf{v} = \mathbf{u} + \mathbf{w}, \tag{8}$$

$$\nabla\mathbf{v}N_d \equiv N_d\nabla\mathbf{v} + \mathbf{v}\nabla N_d = 0, \quad N = N_d + N_v, \tag{9}$$

where N_v, N_d and N are the molar densities of water vapor, dry air constituents and moist air as a whole, respectively. Air velocity $\mathbf{v}$ is equal to the sum of the horizontal $\mathbf{u}$ and vertical $\mathbf{w}$ velocity components. The quantity S ($\mathrm{mol\,m^{-3}\,s^{-1}}$) represents volume density of the rate at which molar density N_v of water vapor is changed by phase transitions. By multiplying (9) by $\gamma_d \equiv N_v/N_d$ and excluding $N_v\nabla\mathbf{v}$ from (8), we obtain

$$\mathbf{v}(\nabla N_v - \gamma_d\nabla N_d) = -S. \tag{10}$$

Using the ideal gas equation of state (see Eq. (1)):

$$p = NRT, \qquad p_v = N_vRT, \qquad p_d = N_dRT, \qquad \gamma_d = \frac{p_v}{p_d}, \tag{11}$$

it is possible to replace molar densities N_i in (10) with partial pressures p_i ($i = v, d$) and the rate of phase transitions S with the power of phase transitions s:

$$\mathbf{v}(\nabla p_v - \gamma_d\nabla p_d) = -s, \quad s = SRT. \tag{12}$$

Note that owing to the universality of the gas constant R the temperature gradient ∇T cancels during this procedure and does not appear in the final equation (12). The kinematic continuity relationship (12) should be fulfilled for any s. The magnitude of power s of water vapor phase transitions is not determined by the continuity equation (12). It must be specified independently by using dynamic physical principles [7].

Such a dynamic physical principle is the equality between the power of phase transition s and the power of horizontal circulation:

$$s = -\mathbf{u}\nabla p. \tag{13}$$

Substituting (13) into (12) and taking into account the easily verifiable identity

$$\nabla p_v - \gamma_d\nabla p_d \equiv (1 + \gamma_d)(\nabla p_v - \gamma\nabla p), \quad \gamma \equiv \frac{\gamma_d}{1 + \gamma_d}, \tag{14}$$

we obtain the following relationship for (12):

$$(\mathbf{w} + \mathbf{u})(\nabla p_v - \gamma\nabla p) = \frac{1}{1 + \gamma_d}\mathbf{u}\nabla p.$$

By transferring $\gamma\mathbf{u}\nabla p$ to the right-hand side of the last relationship and taking into account relationship (14) between γ and γ_d, we obtain: