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Composite system in noncommutative space and the equivalence principle

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ABSTRACT

The motion of a composite system made of N particles is examined in a space with a canonical noncommutative algebra of coordinates. It is found that the coordinates of the center-of-mass position satisfy noncommutative algebra with effective parameter. Therefore, the upper bound of the parameter of noncommutativity is re-examined. We conclude that the weak equivalence principle is violated in the case of a non-uniform gravitational field and propose the condition for the recovery of this principle in noncommutative space. Furthermore, the same condition is derived from the independence of kinetic energy on the composition.

1. Introduction

Recently, noncommutativity has received much attention owing to the development of String Theory [1,2] and Quantum Gravity [3]. The idea that space might have a noncommutative structure has a long history. It was proposed by Heisenberg and was formalized by Snyder [4].

The noncommutative space can be realized as a space where the coordinate operators satisfy the following commutation relations

$$[\hat{X}_i, \hat{X}_j] = i\hbar\theta_{ij}, \tag{1}$$

where θ_{ij} is a constant antisymmetric object. In the classical limit $\hbar \to 0$ the quantum-mechanical commutator is replaced by the Poisson bracket

$$\{X_i, X_j\} = \theta_{ij}. \tag{2}$$

It is important to note that a charged and massive particle in a strong magnetic field $\mathbf{B}$ pointing in the Z direction moves in a noncommutative space. The commutation relation for the coordinates of a particle moving in the (X, Y) plane is given by

$$[\hat{X}, \hat{Y}] = -i\hbar\frac{c}{eB}, \tag{3}$$

here e is the charge of the particle and c is the speed of light [5].

Many physical problems have been studied in the framework of noncommutative quantum mechanics and noncommutative classical mechanics. Some of the first articles on quantum mechanics with noncommutativity of canonical type are [6–9]. Formal aspects of noncommutative quantum mechanics are addressed in [10,11]. Neutrons in a gravitational field with noncommutativity are considered in [12]. Interesting effects arise when one considers noncommutativity in the context of quantum cosmology and black hole physics [13–15]. The Landau problem [16–20], harmonic oscillator [21–24], two-dimensional system in central potential [6], classical particle in a gravitational potential [25,26], classical systems with various potentials [27] are studied. Note, however, that it is important to consider many-particle problem in order to analyze the properties of a wide class of physical systems in noncommutative space.

The classical problem of many particles in noncommutative space–time was examined in [28]. The authors considered two examples of many-particle systems, namely the set of N interacting harmonic oscillators and the system of N particles moving in the gravitational field. The corresponding Newton equation for each particle in these systems was provided. In [29] the two-body system of particles interacting through the harmonic oscillator potential was considered on a noncommutative plane. The authors implemented the noncommutativity through defining a new set of commutating coordinates and got the θ-dependent Hamiltonian in usual commutative plane. The coordinates of the center-of-mass position and relative motion, the total momentum and the relative momentum were introduced in the traditional way. Therefore, the authors rewrote the Hamiltonian as a sum of the freely moving part and a θ-dependent bounded term and derived the partition

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function of a two-body system of classical noncommutative harmonic oscillator.

The problems of noncommutative multiparticle quantum mechanics are examined in [30]. The authors considered the case when the particles of opposite charges feel opposite noncommutativity. The coordinates of the center-of-mass and relative motion were introduced. It was shown that the magnitude of the center-of-mass coordinates noncommutativity is never large then the parameter of noncommutativity for elementary particle. In [31] a system of two quantum particles was considered in the context of noncommutative quantum mechanics, characterized by noncommutativity between the coordinates and momentum noncommutativity. The noncommutative correction to the energy spectrum of two-particle system was found. In [32] the system of two charged quantum particles was considered in a space with coordinates noncommutativity. The authors reduced the two-body problem to a one-body problem for the internal motion. The quantum model of many particles moving in twisted N-enlarged Newton–Hooke space–time was proposed in [33]. The Schroedinger equation for arbitrary stationary potential was provided. As an example the author examined the system of N particles moving "in" and interacting "by" the Coulomb potential.

In the case of Doubly Special Relativity the problem of composite system (so-called soccer-ball problem) was considered in [34–36]. This problem was also studied within the framework of relative locality in [37–39].

Composite system in deformed space with minimal length $[\hat{X},\hat{P}] = i\hbar(1+\beta\hat{P}^2)$ was considered in [40]. The authors solved the two-body problem, studied the composite system made of N elementary particles, defined an effective deformation parameter and re-examined the estimation of the minimal length upper bound. In [41] the properties of the kinetic energy of a composite body were analyzed. The author considered the problem of violation of the equivalence principle and proposed the way to recover this principle in deformed space with minimal length.

The violation of the equivalence principle is an important problem in noncommutative space. In [42] the authors examined the free-fall of a quantum particle in a uniform gravitational field. It was argued that the equivalence principle extends to the realm of noncommutative quantum mechanics. One of the consequences of the twisted Poincare symmetry was investigated in [43]. In this context the author concluded that one can expect that the equivalence principle is not violated in the noncommutative space–time. However, in [44–46], the authors argued that noncommutativity leads to an apparent violation of the equivalence principle.

In this Letter the two-particle and N-particle systems are examined in noncommutative space. We consider the general case when the different particles satisfy noncommutative algebra with different parameters of noncommutativity. Every macroscopic body consist of elementary particles which feel the effect of noncommutativity with different parameters. So, there is a problem of describing the motion of the center-of-mass of macroscopic body in noncommutative space. In order to solve this problem the total momentum is introduced as an integral of motion in noncommutative space and the center-of-mass position is found as its conjugate variable. We conclude that the coordinates of the center-of-mass satisfy noncommutative algebra with effective parameter of noncommutativity. Taking into account this conclusion the condition to recover the equivalence principle in noncommutative space is proposed. Moreover, the same condition is derived from the independence of kinetic energy on the composition.

This Letter is organized as follows. In Section 2 the two-body problem is solved. More general case of composite system made of N elementary particles in noncommutative space is studied in Section 3. The motion of a body in gravitational field and the equivalence principle are considered in Section 4. The properties of the kinetic energy in noncommutative space are studied in Section 5. In Section 6 the upper bound of the parameter of noncommutativity is re-examined.

2. Two-body problem

In ordinary space we can reduce a two-body problem to an equivalent one-body problem. Let us consider two elementary particles of masses m_1, m_2 that interact only with each other in two-dimensional noncommutative space and define the total momentum and the center-of-mass position of this system. We consider the case when the different particles of masses m_1, m_2 satisfy the noncommutative algebra with parameters θ_1, θ_2 respectively. Therefore, the coordinates $X_\mu^{(i)}$ and the components of momentum $P_\mu^{(i)}$ satisfy the following relations

$$\{X_1^{(i)}, X_2^{(j)}\} = -\{X_2^{(i)}, X_1^{(j)}\} = \delta^{ij}\theta_i, \tag{4}$$

$$\{X_\mu^{(i)}, P_\nu^{(j)}\} = \delta_{\mu\nu}\delta^{ij}, \tag{5}$$

$$\{P_\mu^{(i)}, P_\nu^{(j)}\} = 0, \tag{6}$$

here $\mu = (1,2)$, $\nu = (1,2)$ and the indices i, j label the particles.

The interaction potential energy of the two particles $V(|\mathbf{X}^{(1)} - \mathbf{X}^{(2)}|)$ depends on the distance between them. Therefore, the Hamiltonian of the system reads

$$H = \frac{(\mathbf{P}^{(1)})^2}{2m_1} + \frac{(\mathbf{P}^{(2)})^2}{2m_2} + V\left(|\mathbf{X}^{(1)} - \mathbf{X}^{(2)}|\right). \tag{7}$$

Let us introduce the total momentum in a traditional way

$$\tilde{\mathbf{P}} = \mathbf{P}^{(1)} + \mathbf{P}^{(2)}. \tag{8}$$

It is easy to find that the total momentum satisfies the following relation

$$\{\tilde{\mathbf{P}}, H\} = 0. \tag{9}$$

So, the total momentum (8) is an integral of motion in noncommutative space. Now we can find the coordinates of the center-of-mass $\tilde{\mathbf{X}}$ as conjugate coordinates to the total momentum

$$\tilde{\mathbf{X}} = \frac{m_1\mathbf{X}^{(1)} + m_2\mathbf{X}^{(2)}}{m_1+m_2}. \tag{10}$$

We can also introduce the coordinates and momentum of relative motion in the traditional way

$$\Delta\mathbf{P} = \mu_1\mathbf{P}^{(2)} - \mu_2\mathbf{P}^{(1)}, \tag{11}$$

$$\Delta\mathbf{X} = \mathbf{X}^{(2)} - \mathbf{X}^{(1)}, \tag{12}$$

here $\mu_1 = m_1/(m_1+m_2)$ and $\mu_2 = m_2/(m_1+m_2)$.

It is easy to find that

$$\{\tilde{X}_\mu, \tilde{P}_\nu\} = \{\mu_1 X_\mu^{(1)} + \mu_2 X_\mu^{(2)}, P_\nu^{(1)} + P_\nu^{(2)}\} = \delta_{\mu\nu}, \tag{13}$$

$$\{\Delta X_\mu, \Delta P_\nu\} = \{X_\mu^{(2)} - X_\mu^{(1)}, \mu_1 P_\nu^{(2)} - \mu_2 P_\nu^{(1)}\} = \delta_{\mu\nu}, \tag{14}$$

$$\{\tilde{P}_\mu, \tilde{P}_\nu\} = \{\Delta P_\mu, \Delta P_\nu\} = 0. \tag{15}$$

Let us calculate the Poisson brackets for the coordinates of the center-of-mass

$$\begin{aligned}\{\tilde{X}_1, \tilde{X}_2\} &= -\{\tilde{X}_2, \tilde{X}_1\} \\ &= \frac{1}{(m_1+m_2)^2}\{m_1X_1^{(1)} + m_2X_1^{(2)}, m_1X_2^{(1)} + m_2X_2^{(2)}\} \\ &= \frac{m_1^2\theta_1 + m_2^2\theta_2}{(m_1+m_2)^2}.\end{aligned} \tag{16}$$