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# Coherent control via interplay between driving field and two-body interaction in a double well

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ABSTRACT

We investigate interplay between external field and interatomic interaction and its applications to coherent control of quantum tunneling for two repulsive bosons confined in a high-frequency driven double well. By using a full solution which is generated analytically as a coherent non-Floquet state, three kinds of the stationary-like states (SLSs) with different degeneracies are illustrated, which corresponding to the different coherent destructions of tunneling (CDT) at the Floquet level-crossing, avoided-crossing and uncrossing points. The analytical results are numerically confirmed and perfect agreements are found. Based on the results, a useful scheme of quantum tunneling switch between the SLSs is presented.

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## 1. Introduction

Coherent control of quantum tunneling in a periodically driven double well has been researched extensively from both theoretical and experimental sides [1–3]. Some interesting phenomena, such as the coherent destruction or coherent construction of tunneling (CDT or CCT) [3–9], chaos enhancing tunneling [10,11], and photon-assisted tunneling [12–14] have been found. The control method and CDT mechanism have also been applied to different fields, such as the qubit control [15,16], the frequency-dependent transistor [17] and the probes of spectrum structures in different systems [18,19].

Many works focus on different routes to CDT for the single- or many-particle systems [4–9,20]. Few-particle systems are in an intermediate case for the numbers of particles, which deserve further study for comprehensively understanding tunneling dynamics. However, investigations on quantum control to two particles in a periodically driven double well are extremely rare, except for the cases of two particles in a one-dimensional lattice which can be reduced to a two-site trap [21–23] or two particles in a driven double-well train [24–27]. Recently, several interesting phenomena of quantum tunneling were shown for two repulsive bosons in a non-driven double well, which include the Rabi oscillations and the correlated pair tunneling [28–30]. The interatomic interaction adjusted by the Feshbach-resonance technique [31] plays an important role in tunneling dynamics of the non-driven two-particle system. Here we are interested in the combined effects of the driving and interaction on tunneling dynamics of the double-well coupled two bosons.

In this Letter, we study coherent control of quantum tunneling via the competition and cooperation between atomic interaction and driving field for a pair of repulsive bosons confined in a periodically driven double well. In the high-frequency regime, we obtain a general non-Floquet state as a full solution, which is a coherent superposition of the Floquet states. It is demonstrated that the photon resonance [21] of interaction leads to translation of the Floquet level-crossing points, and the non-resonant interaction causes avoided crossing of partial levels. Three different kinds of CDT, respectively at the level-crossing points for the three degenerate states, at the avoided-crossing points for the two degenerate states and at the uncrossing points for the single Floquet states, are illustrated, which result in different kinds of the stationary-like states (SLSs) with invariant populations. The analytical results are confirmed by direct numerical simulations and good agreements are shown. As an application of the above results, an interesting scheme of quantum tunneling switch between SLSs is presented by using CDT and CCT to close and open the quantum tunneling, which could be useful for the quantum control of two bosons in a driven double well.

## 2. Analytical solutions in high-frequency regime

We consider two bosons confined in a periodically driven double well with the governing Hamiltonian [32,33]

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$$H(t) = \frac{\varepsilon(t)}{2}\left(b_1^\dagger b_1 - b_2^\dagger b_2\right) + \gamma\left(b_1^\dagger b_2 + b_2^\dagger b_1\right) + \frac{U}{2}\left[b_1^\dagger b_1\left(b_1^\dagger b_1 - 1\right) + b_2^\dagger b_2\left(b_2^\dagger b_2 - 1\right)\right], \tag{1}$$

where $b_i(b_i^\dagger)$ is the annihilation (creation) operator for $i$th well. The parameters $\gamma$ and $U$ are the tunneling coefficient and the interaction strength, respectively. The function $\varepsilon(t) = \varepsilon_0\cos(\omega t)$ describes the external *ac* field in which $\omega$ and $\varepsilon_0$ are the driving frequency and amplitude, respectively. For simplicity, we have put $\hbar = 1$ and taken the reference frequency $\omega_0 = 10^2 s^{-1}$ [32] so that $\varepsilon_0$, $\gamma$, $\omega$ and $U$ are in units of $\omega_0$, and time $t$ is normalized in units of $\omega_0^{-1}$.

Quantum state $|\psi(t)\rangle$ of system (1) can be expanded in Fock bases as [21]

$$\left|\psi(t)\right\rangle = a_0(t)|0,2\rangle + a_1(t)|1,1\rangle + a_2(t)|2,0\rangle, \tag{2}$$

where the state vectors $|j\rangle = |j, 2-j\rangle (j = 0, 1, 2)$ denote that $j$ bosons reside in left well, $2-j$ bosons reside in right well; $a_j(t)$ represents probability amplitude of the system in $j$-th Fock state $|j\rangle$. The corresponding normalization condition reads as $|a_0(t)|^2 + |a_1(t)|^2 + |a_2(t)|^2 = 1$. Inserting Eqs. (1) and (2) into the time-dependent Schrödinger equation $i\frac{\partial|\psi(t)\rangle}{\partial t} = H(t)|\psi(t)\rangle$ results in the coupling equations

$$\begin{aligned} i\dot{a}_0(t) &= \left[U - \varepsilon(t)\right]a_0(t) + \sqrt{2}\gamma a_1(t), \\ i\dot{a}_1(t) &= \sqrt{2}\gamma a_0(t) + \sqrt{2}\gamma a_2(t), \\ i\dot{a}_2(t) &= \left[U + \varepsilon(t)\right]a_2(t) + \sqrt{2}\gamma a_1(t). \end{aligned} \tag{3}$$

It is difficult to get exact analytical solutions of Eq. (3) with periodic driving $\varepsilon(t) = \varepsilon_0\cos(\omega t)$. However, we can construct the approximate analytical solution in the high-frequency limit, $\omega \gg 1$ and $u, \gamma \ll \omega$. To do this, we adopt the idea of reduced interaction strength [21] to rewrite the interaction strength as $U = n\omega + u$ for $0 \leqslant u \ll \omega$, $n = 0, 1, 2, \ldots$ with $u$ being the reduced interaction strength, and make the function transformation $a_0(t) = b_0(t)e^{i\int[\varepsilon(t)-n\omega]\,dt}$, $a_1(t) = b_1(t)$, $a_2(t) = b_2(t)e^{-i\int[\varepsilon(t)+n\omega]\,dt}$ with $b_j(t)(j = 0, 1, 2)$ being the slowly-varying functions of time. Thus Eq. (3) becomes new equations in terms of $b_j(t)$. Under the high-frequency approximation, the rapidly oscillating functions $e^{i\int[\pm\varepsilon(t)-n\omega]\,dt} = \sum_{n'}\mathcal{J}_{n'}(\pm\frac{\varepsilon_0}{\omega})e^{i(n'-n)\omega t}$ in the equations of $b_j(t)$ can be replaced by their time averages [34] $\mathcal{J}_n(\pm\frac{\varepsilon_0}{\omega})$ which are the $n$-order Bessel functions such that the final equations become

$$\begin{aligned} i\dot{b}_0(t) &= ub_0(t) + J_n\left(\frac{\varepsilon_0}{\omega}\right)b_1(t), \\ i\dot{b}_1(t) &= J_n\left(\frac{\varepsilon_0}{\omega}\right)b_0(t) + (-1)^n J_n\left(\frac{\varepsilon_0}{\omega}\right)b_2(t), \\ i\dot{b}_2(t) &= ub_2(t) + (-1)^n J_n\left(\frac{\varepsilon_0}{\omega}\right)b_1(t). \end{aligned} \tag{4}$$

In the calculations, we have employed the formula $\mathcal{J}_n(-\frac{\varepsilon_0}{\omega}) = (-1)^n\mathcal{J}_n(\frac{\varepsilon_0}{\omega})$ for positive integer $n$, and the renormalized coupling coefficient $J_n(\frac{\varepsilon_0}{\omega}) = \sqrt{2}\gamma\mathcal{J}_n(\frac{\varepsilon_0}{\omega})$. Starting from Eq. (4), we obtain the interesting analytical solutions as follows.

### 2.1. Quasienergies and Floquet states

Because the time-dependent Hamiltonian (1) has the period $T = \frac{2\pi}{\omega}$, we can make use of Floquet theory [35] to get its solution in the form $|\psi(t)\rangle = e^{-iEt}|\varphi(t)\rangle$, where $|\varphi(t)\rangle$ is the Floquet state with the same period $\frac{2\pi}{\omega}$, $E$ is termed the Floquet quasienergy which has been normalized in units of $\hbar\omega_0$. Noting the same period of the transformation function $e^{i\int[\pm\varepsilon(t)-n\omega]\,dt}$ between $a_j(t)$ and $b_j(t)$, to generate the Floquet states, we seek the stationary solutions of Eq. (4) $b_0 = Ae^{-iEt}$, $b_1 = Be^{-iEt}$, $b_2 = Ce^{-iEt}$ with constants $A$, $B$, $C$ obeying the normalization condition $|A|^2 + |B|^2 + |C|^2 = 1$. Inserting these into Eq. (4), we establish the equations of the stationary solutions. By solving these equations, we obtain three Floquet quasienergies $E_l$ and three sets of constants $A_l$, $B_l$, $C_l$ as

$$E_0 = u, \qquad A_0 = \frac{1}{\sqrt{2}}, \qquad B_0 = 0, \qquad C_0 = (-1)^{n+1}\frac{1}{\sqrt{2}};$$

$$E_1 = \frac{1}{2}(u - k_n), \qquad A_1 = \frac{2J_n}{\sqrt{8J_n^2 + (u+k_n)^2}}, \tag{5}$$

$$B_1 = \frac{-(u+k_n)}{\sqrt{8J_n^2 + (u+k_n)^2}}, \qquad C_1 = \frac{(-1)^n 2J_n}{\sqrt{8J_n^2 + (u+k_n)^2}};$$

$$E_2 = \frac{1}{2}(u + k_n), \qquad A_2 = \frac{2J_n}{\sqrt{8J_n^2 + (k_n-u)^2}}, \tag{6}$$

$$B_2 = \frac{k_n - u}{\sqrt{8J_n^2 + (k_n-u)^2}}, \qquad C_2 = \frac{(-1)^n 2J_n}{\sqrt{8J_n^2 + (k_n-u)^2}}. \tag{7}$$

Here, the simplified parameter $k_n = \sqrt{8J_n^2(\frac{\varepsilon_0}{\omega}) + u^2}$ has been adopted. Given $b_j(t)$, the rapidly oscillating function $a_j(t)$ are obtained immediately. Substituting such $a_j(t)$ into Eq. (2), Floquet states $|\psi_l(t)\rangle$ are expressed as

$$\left|\psi_l(t)\right\rangle = e^{-iE_lt}\left|\varphi_l(t)\right\rangle = e^{-iE_lt}\left[A_l e^{\frac{i\varepsilon_0}{\omega}\sin(\omega t)-in\omega t}|0,2\rangle + B_l|1,1\rangle + C_l e^{\frac{-i\varepsilon_0}{\omega}\sin(\omega t)-in\omega t}|2,0\rangle\right] \tag{8}$$

for $l = 0, 1, 2$. Here $n$ is a positive integer adjusted by the interaction strength. If the system is prepared in the Floquet states initially, probabilities of the system in different Fock states are the constants $|A_l|^2$, $|B_l|^2$ and $|C_l|^2$, respectively, so that tunneling of atoms between two wells is suppressed completely, i.e., a kind of CDT occurs.

### 2.2. Coherent non-Floquet states

Given the Floquet solutions of Eqs. (5)–(8) as a set of complete bases in a three-dimensional Hilbert space [29,36], the principle of superposition of quantum mechanics indicates that the linear Schrödinger equation has the periodic or quasiperiodic non-Floquet solutions, which is a coherent superposition of the Floquet states. Noticing the relation between $a_j(t)$ and $b_j(t)$, the general non-Floquet state has the form

$$\begin{aligned} \left|\psi(t)\right\rangle &= \sum_{l=0}^{2} c_l\left|\psi_l(t)\right\rangle = a_0'(t)|0,2\rangle + a_1'(t)|1,1\rangle + a_2'(t)|2,0\rangle \\ &= b_0'(t)e^{\frac{i\varepsilon_0}{\omega}\sin(\omega t)-in\omega t}|0,2\rangle + b_1'(t)|1,1\rangle \\ &\quad + b_2'(t)e^{-\frac{i\varepsilon_0}{\omega}\sin(\omega t)-in\omega t}|2,0\rangle, \end{aligned} \tag{9}$$

where $c_l$ are the superposition coefficients determined by the initial conditions and normalization, and $b_j'(t)$ are the slowly varying functions

$$b_0'(t) = \sum_{l=0}^{2} c_l A_l e^{-iE_lt}, \tag{10}$$

$$b_1'(t) = \sum_{l=0}^{2} c_l B_l e^{-iE_lt}, \tag{11}$$

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