Physics Letters A 377 (2013) 3112–3118

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Minimum energy control and optimal-satisfactory control of Boolean control network

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ARTICLE INFO

Article history:
Received 6 April 2013
Received in revised form 23 September 2013
Accepted 2 October 2013
Available online 7 October 2013
Communicated by A.R. Bishop

Keywords:
Boolean control network
Minimum energy control
Optimal-satisfactory control
Semi-tensor product

ABSTRACT

In the literatures, to transfer the Boolean control network from the initial state to the desired state, the expenditure of energy has been rarely considered. Motivated by this, this Letter investigates the minimum energy control and optimal-satisfactory control of Boolean control network. Based on the semi-tensor product of matrices and Floyd's algorithm, minimum energy, constrained minimum energy and optimal-satisfactory control design for Boolean control network are given respectively. A numerical example is presented to illustrate the efficiency of the obtained results.

1. Introduction

Boolean network was first proposed by Kauffman to describe genetic regulatory networks. In a Boolean network, the states of Boolean variables correspond to the activities of genes. The state of each Boolean variable is described by a binary variable, where the value 1 (or 0) represents the Boolean variable is on (or off). Moreover, every Boolean variable updates its value according to a logical relationship, given in the form of a Boolean function. As Boolean network is a powerful tool in describing, analyzing and simulating the cellular network, it has received much attention. Up to now, many results on the topological structure of a Boolean network have been presented, see e.g. [1–3]. There is also a huge amount about Boolean networks in literature, for example, see [4].

Recently, a new matrix product called semi-tensor product is emerging, which has a wide range of application in many fields, for example, see [5–8]. The semi-tensor product of matrices has been successfully applied to express and analyze Boolean networks. A set of useful formulas for calculating fixed points and cycles of Boolean networks was given by [5]. Many fundamental and landmark results about Boolean control networks have been obtained by this method. For example, the controllability and observability of BCN have been studied by [9], and the synchronization has been investigated by [10].

Optimal control theory is a mathematical optimization method for deriving control polices, see e.g. [11–13]. The optimal control of Boolean networks has attracted attention too. For example, by using a maximum principle, [14] has discussed the Mayer-type optimal control for single-input Boolean networks, that is fix a final time $N>0$, maximize the cost-functional $J(u)=r^T x(N;u)$. [15] has studied the problem of minimizing the cost functional $J(u)=\lim_{M\to\infty}\frac{1}{M}\sum_{t=0}^{M-1}P(x(t),u(t))$ of k-valued logical system (when $k=2$, k-valued logical system degenerates into a Boolean network), that is the average-cost per-stage infinite-horizon problem. Time-optimal control problem of Boolean networks has been studied too, [16]. Though the optimal control of Boolean networks has been studied, and some research results have been reported in the literature, we can note that the minimum energy control of minimum energy control and satisfactory control of Boolean networks are still lacking in the literature. The minimum energy control and satisfactory control are meaningful. Optimal control deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. Energy constraints are also encountered naturally in practical systems, because any physical system can only be powered with finite energy [17]. In optimal control theory, the minimum energy control is one of the most fundamental and important problems in modern control, for example, see [18,19]. The minimum energy control is the control $u(t)$ that steers a system to a desired state with a minimum expenditure of energy. Satisfactory control was proposed aiming at the optimal control of sophisticated industry process. Its primary principle is that the solving of optimal control of sophisticated industry process can be considered as a procedure of obtaining the

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0375-9601/$ – see front matter
http://dx.doi.org/10.1016/j.physleta.2013.10.002

satisfactory solution with the adjustment of the operator [20]. Satisfactory control has been widely discussed in the literature, see e.g. [21]. But there have been no results on the minimum energy control and satisfactory control of Boolean network to the best of our knowledge.

Motivated by the above, in this Letter, we study the minimum energy control and satisfactory control of a Boolean network. We want to find a control sequence that transfers a Boolean network from the initial state $x(0) = x_0$ to the desired state $x_d = x(s)$ and minimizes the energy consumption $P(u) = \sum_{t=0}^{s-1} u^T(t)Qu(t)$. Moreover, we want to solve the satisfactory control problem of a Boolean network. That is, we will choose a control sequence to ensure the performance index $P(u) = \sum_{t=0}^{s-1} u^T(t)Qu(t)$ is not more than a prescribed value a. In general, it is possible to superimpose other requirements like the requirement of the minimum time from the initial state to the desired state, i.e. the optimal-satisfactory control problem [22]. Hence, we consider the minimum time optimization problem, and $P(u) = \sum_{t=0}^{s-1} u^T(t)Qu(t) \leqslant a$ as an inequality constraint. That is, we will find the minimum time s from the initial state x_0 to the desired state $x_d = x(s)$, s.t. $P(u) = \sum_{t=0}^{s-1} u^T(t)Qu(t) \leqslant a$. The main tools in this Letter are semi-tensor product of matrices and Floyd's algorithm. Floyd's algorithm is a famous algorithm for finding shortest paths in a weighted graph. Although there are some other algorithms for finding the shortest paths, such as Dijkstra's algorithm and Johnson's algorithm, Floyd's algorithm has its own advantages. For example, Dijkstra's algorithm needs the graph to have a single source and the edge weights to be nonnegative, Johnson's algorithm can only be applied to sparse directed graphs, Floyd's algorithm has no such limitations [15,23]. The main contributions of this Letter are as follows. (i) The minimum energy control of Boolean network is studied for the first time by using the semi-tensor product, to the best of our knowledge. (ii) Algorithms are provided to transfer the system from the initial state to the desired state while avoiding unfavorable states with minimum energy. (iii) The concepts of optimal satisfactory is given in this Letter, and an algorithm is presented to solve this problem.

The rest of this Letter is organized as follows. Section 2 gives the preliminaries. In Section 3, we first solve the minimum energy control problem of the BCN. Algorithms are given to transfer the BCN from the initial state to the desired state with the minimum expenditure of energy. Next, the constrained minimum energy control design is given. That is to solve the minimum energy control problem of the BCN while avoiding a set of forbidden states. Section 4 studies the optimal-satisfactory problem of the BCN, that is to study the minimum time optimization problem and the expenditure of energy is not more than a prescribed value. Section 5 gives the illustrative example, which is followed by the conclusion in Section 6.

2. Preliminaries

In this section, we will review the semi-tensor product (STP) and Floyd's algorithm.

2.1. Semi-tensor product

In this subsection, we will introduce the semi-tensor product and some concerning results.

Definition 2.1. (See [24].) For $M \in \mathbb{R}^{m\times n}$ and $N \in \mathbb{R}^{p\times q}$, their STP (also called Cheng product), denoted by $M \ltimes N$, is defined as follows:

$$M \ltimes N := (M \otimes I_{s/n})(N \otimes I_{s/p}),$$

where s is the least common multiple of n and p, and $\otimes$ is the Kronecker product.

Remark 2.1. The matrix product we use in this Letter is the semi-tensor product (STP). Since STP is a generalization of conventional matrix product, we omit the symbol "$\ltimes$", if no confusion raises.

Table 1
Some notations.

Notation	Explanation
$\mathcal{M}_{m\times n}$	The set of all $m \times n$ matrices
Δ_k	$\Delta_k := \{\delta_k^i \mid i = 1, 2, \dots, k\}$, where δ_k^i is the ith column of the identity matrix I_k
$\mathcal{L}_{n\times s}$	A matrix $A \in \mathcal{M}_{m\times n}$ is called a logical matrix, if the columns of A are elements of Δ_m. The set of $n \times s$ logical matrices is defined by $\mathcal{L}_{n\times s}$
$\mathrm{Blk}_i(A)$	The ith $n \times n$ square block of a matrix $A \in \mathcal{M}_{n\times mn}$, $i = 1, 2, \dots, m$
$A_{i,j}$	The ith element of the jth column of a matrix A

Next, we give some other notations, which are collected in Table 1.

Letting $\mathcal{D} := \{1, 0\}$. By identifying $T = 1 \sim \delta_2^1$, $F = 0 \sim \delta_2^2$, then the logical variable $A(t)$ takes value from these two vectors, i.e. $A(t) \in \Delta := \Delta_2 = \{\delta_2^1, \delta_2^2\}$.

The following lemma is fundamental for the matrix expression of the logical function.

Lemma 2.1. *(See [24].) Any logical function $L(A_1, \dots A_n)$ with logical arguments $A_1, \dots, A_n \in \Delta$ can be expressed in a multi-linear form as*

$$L(A_1, \dots A_n) = M_L A_1 A_2 \cdots A_n,$$

where $M_L \in \mathcal{L}_{2\times 2^n}$ is unique, called the structure matrix of L.

2.2. Floyd's algorithm

In this subsection, we will review Floyd's algorithm [25,26], which will be used in the following discussion.

A graph G is a pair $G = (V, E)$, where V is a finite set of nodes or vertices, and E has as elements subset of V of cardinality two called edges. A directed graph is a graph with directions assigned to its edges. A weighted directed graph is $G = (V, E)$ together with a function w from E to Z (usually just Z^+; it can also be R^+ when, for example the weights are Euclidean distances). In certain cases, we shall use more mnemonic name for weights, such as c (for cost) or d (for distances).

For a weighted directed graph $G = (V, E)$, where the vertices $V = \{1, 2, \dots, N\}$, denote by c_{ij} the weight of the edge (i, j). Set

$$d_{ij}(0) = \begin{cases} c_{ij}, & (i, j) \in E \text{ and } i \neq j, \\ \infty, & \text{otherwise.} \end{cases}$$

The Floyd's algorithm is: For $\alpha = 1, \dots, N$, $i = 1, \dots, N$, $i \neq \alpha$, $j = 1, \dots, N$, $j \neq \alpha$, iteratively calculate

$$d_{ij}(\alpha) = \min\{d_{ij}(\alpha - 1), d_{i\alpha}(\alpha - 1) + d_{\alpha j}(\alpha - 1)\}.$$

Assume there are no negative cycles, that is, there are no cycles of negative length, then $d_{ij}(N)$ is the sum of weights through the shortest path from vertex i to vertex j. $d_{ij}(\alpha)$ is the sum of weights through the shortest path from vertex i to vertex j which only passes some nodes contained in $\{1, 2, \dots, \alpha\}$.

3. Main results

A Boolean control network with n variables and m inputs is described as