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Construction of Arnold tongue structures for coupled periodic oscillators

Fernando Montoya [a], M. Rivera [a,*], J. Escalona [a], P. Parmananda [b]

[a] Facultad de Ciencias, UAEM, Avenida Universidad 1001, Colonia Chamilpa, Cuernavaca, Morelos, Mexico
[b] Department of Physics, Indian Institute of Technology, Bombay, Powai, Mumbai – 400 076, India

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ABSTRACT

Arnold tongue structures generated due to the mutual entrainment of two periodic oscillators are studied experimentally and numerically. This mutual entrainment is provoked due to the mutual (bidirectional) coupling between the two oscillators. In experiments, this bidirectional coupling is achieved by immersing a pair of anodes (oscillators) in a common electrolytic solution. A voltage mismatch between these anodes renders the time period of the uncoupled oscillators non-identical. Moreover, the coupling strength between the two oscillators is uniquely determined by the Euclidean distance separating them. Systematically varying the distance between these two anodes as a function of their voltage mismatch, phase locked domains were located. Subsequently, Arnold tongue structures were constructed in the experiments. Numerical simulations, using a model for electrochemical corrosion, corroborate our experimental findings.

1. Introduction

Mutual coupling of two periodic oscillators is generally studied in the context of the synchronization phenomena. For example, the first reported experiments involving synchronization of periodic oscillators were performed by Christian Huygens [1] in the 17th century. In these experiments, two clock pendulums were mutually coupled through a common supporting beam. This field of synchronization was subsequently extended to include the synchronization of chaotic oscillations [2–4]. Since then, interest in this area of research has avalanched attracting researchers from diverse fields such as physics [5], chemistry [6] and biology [7].

Another studied aspect, involving the mutual (bidirectional) coupling of two oscillators, is the phenomena of mutual entrainment. This mutual entrainment can provoke a plethora of different dynamical behaviors ranging from harmonic, quasiperiodic and chaotic responses in the coupled system. Similar regimes of dynamical behaviors have also been observed when a single oscillator, executing period one dynamics, is subjected to harmonic forcing. In particular, this forcing of the periodic oscillator leads to the emergence of phase locked domains separated by regions in the parameter space where quasiperiodic behavior is observed. Usually, for this scenario, a systematic scan of the parameter space reveals the existence of Arnold tongue structures [8,9].

Mutual entrainment has been studied theoretically [10] and its emergence in electrochemical experiments has been predicted using phase models [11]. A characteristic feature of entrainment in nonlinear systems is the emergence of the Arnold tongue structures in the parameter space. These structures include phase locked domains in conjunction with regions where quasiperiodic behavior are provoked by virtue of the entrainment phenomena. Arnold tongue structures have been reported for electrical, hydrodynamical, chemical and biological systems [12–14].

In the present work, we report the existence of Arnold tongue structures in a coupled system by virtue of the mutual entrainment of two period one oscillators. In the next section, experimental results involving mutual entrainment are presented. Similar results have previously been recently reported by Nakabayashi and coworkers [15]. A four electrode electrochemical cell is configured for these experiments. In Section 3, corresponding numerical simulations, that support the experimental observations, are presented. Finally, in Sections 4, a brief summary of the obtained results is outlined.

2. Experimental observations

The experiments were designed to study the effect of mutual coupling on the dynamics of two oscillators. It is important to point out that the individual dynamics for both the oscillators, prior to their coupling, exhibit period one behavior. A four electrode electrochemical cell including two anodes, a common

* Corresponding author. Tel.: +52 777 3297020; fax: +52 777 3297040.
E-mail address: mrivera@uaem.mx (M. Rivera).

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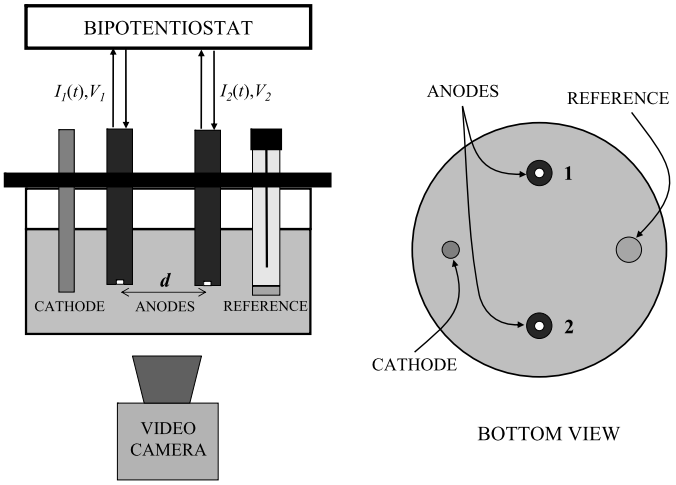

Fig. 1. The figure on the left is the schematic representation of the four electrode electrochemical cell configured to study mutual entrainment of two oscillators. The two iron anodes, a common copper cathode and a common SCE reference electrode are all immersed in the electrolytic solution. The camera, used for recording images, and the bipotentiostat are also shown. The bottom view of the experimental setup, shown in the figure on the right, indicates the relative position of each one of these electrodes.

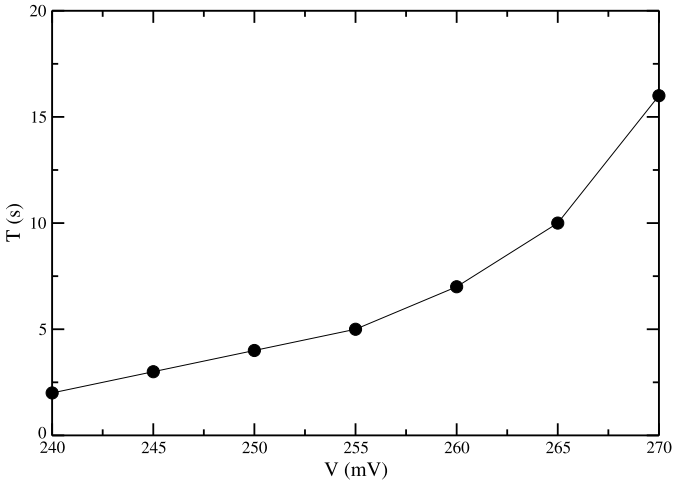

Fig. 2. Time period variations of the period one dynamics for a single oscillator as a function of the superimposed anodic voltage. Only the parameter interval wherein the oscillations are found is illustrated.

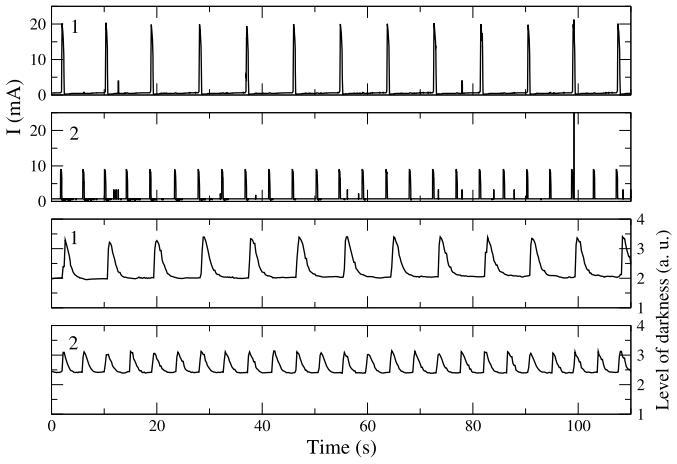

Fig. 3. The top two panels show the time series I_1 and I_2 respectively. Visual inspection reveals that the two time series are phase locked with a ratio of 1:2 due to the underlying mutual coupling. The bottom panels show the analogous results if one records the time series for the level of darkness on the surface of the anodes. The modulations in the level of darkness for the two surfaces also exhibits 1:2 phase-locking. The systems parameters were fixed at $V_1 = 255$ mV, $V_2 = 270$ mV. The Euclidean distance between the two anodes is fixed at $d = 10$ mm.

cathode and a common reference saturated calomel electrode (SCE) was configured. All these four electrodes were immersed in an electrolytic solution. Each anode is an iron disk (Sigma Aldrich, 99.98% purity) of 1 mm diameter encapsulated by epoxy such that the reactions take place only on the surface of the anodes. It is ensured that the immersion depth for both the anodes is identical for symmetry purposes. The measured Euclidean distance corresponds to the separation between the centers of the two disks. The voltages (V_1, V_2) applied to the two anodes (anodic voltage) are measured with respect to a common reference electrode (SCE). These anodic voltages act as the control/bifurcation parameters in the experiments. The common cathode is a copper rod with a diameter of 5 mm and is 2 cm long. The currents flowing between the anodes and the common cathode are the anodic currents (I_1, I_2). These anodic currents are the observables in the experiments. Finally, the electrolytic solution in which the electrodes are immersed is a mixture of 1 M sulfuric acid and 0.4 M cupric sulfate. For our experiments, a volume of 300 ml is maintained in the electrochemical cell. The experiments were carried out at room temperatures ($\sim$ 298 K).

Employing a bipotentiostat (PINE, Model AFRDE5) both the anodic voltages (V_1, V_2) were maintained constant where as the two anodic currents (I_1, I_2) were simultaneously recorded using a 12 bits data acquisition card with a sampling rate of 120 Hz. It needs to be mentioned that in our experiments the least count of the potentiostat is that of 1 mV. This limits our resolution while scanning the anodic voltages. Apart from the analog recording of the anodic currents, the dynamical evolution was also examined using image analysis of the electrode surface. A video camera was utilized to monitor the activity on the surfaces of the two anodes. In particular, the cyclic deposition and the dissolution of a passivating film formed during the corrosion process of the iron electrodes was identified. Fig. 1 shows a schematic of the experimental setup along with the bottom view indicating the location of the four electrodes immersed in the electrolytic solution.

An exhaustive analysis of the autonomous dynamics for a single electrode, including the underlying bifurcations, has already been reported elsewhere [16]. It reveals the existence of an interval in parameter space wherein period one oscillations of anodic current (I) are found. This parameter interval is sandwiched between parameter domains where the anodic current dynamics exhibit fixed point behavior. For the present experiments, the system parameters (V_1, V_2) were chosen such that the anodic currents (I_1, I_2) of the uncoupled anodes exhibit period one oscillations. The time periods of these oscillations can be regulated by varying the applied anodic voltages (V_1, V_2), as shown in Fig. 2.

Fig. 3 shows a (1:2) phase locking of the two oscillators [15] by virtue of their mutual entrainment. The top two panels show this (1:2) phase locking as observed in the time series of the anodic current oscillations (I_1, I_2). The bottom two panels show the same phenomena as viewed by the image analysis. The system parameters are included in the corresponding figure caption. To elaborate more on the image analysis protocol, a zoom of a section of the two time series representing the level of darkness (bottom two panels of Fig. 3) of the electrode surfaces are presented in Fig. 4. Also superimposed in the figure is the corresponding sequence of optical snapshots. The figure describes how the sequence of optical snapshots taken by the camera were converted to a time series of level of darkness in arbitrary units (a.u.) as shown in Fig. 3.

Considering a fixed distance d and fixing the anodic voltage of one of the anodes at around 255 mV (approximately in the mid-