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Metastable state of three-component two-temperature plasma and fireball problem

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ABSTRACT

It has been shown that like-charged particles inside electroneutral plasma can be attracted to each other, in contrast with non-electroneutral plasma considered in the literature, where such particles electrostatically repulse when they have any distance between them. We have calculated an analytical formula for the free energy of electroneutral three-component two-temperature plasma, from the basis of the Poisson–Boltzmann electrostatic equation. It is shown that free energy has a local minimum when the temperature of electrons exceeds 2000 K, when the quantity of electrons is less than 20% of the total quantity of negatively charged particles, when the temperature of ions is 300 K and when the distance between ions is several Debye radii. Plasma of a specified structure, temperature and density can mimic a fireball substance, in contrast with, for example, two-component isothermal plasma which has no minimum free energy at any interparticle distance.

1. Introduction

A fireball is a fascinating phenomenon observed in both nature and laboratory conditions, and has a substance density close to the density of atmospheric air. Fireballs are spherical, with typical radius from millimetres to tens of centimetres. They possess a large internal energy, glow, and have a typical life time from seconds to a number of minutes [1–11]. The nature and mechanism of the formation of fireballs is still not completely clear. There are more than one hundred known models of this phenomenon, and plasma-based models are popular among them. They can explain glow and energy content, but their main drawback consists of an inability to describe the fireball's macroscopic life time without an external supply of energy. The macroscopic life time of plasma is possible at low recombination speeds, as well as with the existence of a special mechanism of substance retention. Without these listed factors, the plasma either recombines, or bounces apart with an inertial time of less than several thousandths of a second. To explain the low recombination speed, the Rydberg idea [12] and non-ideal plasma [13,14] can be considered, but the possibility of spatial retention of the plasma substance isn't clear, and requires further comprehensive study.

The ability to retain the plasma substance assumes the existence of the special mechanism of attraction of like-charged particles in a plasma. In frequently quoted works [15,16], the possibility of electrostatic attraction between like-charged particles put into an electroneutral plasma was studied. The main conclusion of these reports, from the basis of the Poisson–Boltzmann equation, was the default of the attraction forces between particles and, therefore, the presence of repulsion forces at any distance between the particles. This conclusion on the impossibility of electrostatic attraction between the like-charged particles is a serious stimulation to research alternative mechanisms of interparticle interaction, for example, wakefield attraction [17], or the collective long-range attraction arising when particle flows interact in plasma [18–21]. However, it should be noted that the conclusion of the default of the electrostatic attraction only concerns the charges of the non-electroneutral systems considered in [15,16]. In order for the system to be electroneutral (the most common situation required by the experiments), it is necessary to place not only the like-charged test particles into the initial neutral plasma, but also charge carriers that compensate the charge of the test particles. The results obtained from systems where estimation of the electroneutrality is conducted in this way may fundamentally differ from the results for systems where the condition of electroneutrality isn't fulfilled.

This report shows, on the basis of the analytical solution of the Poisson–Boltzmann equation, the possibility of long-range electrostatic attraction inside the electroneutral system of charges.

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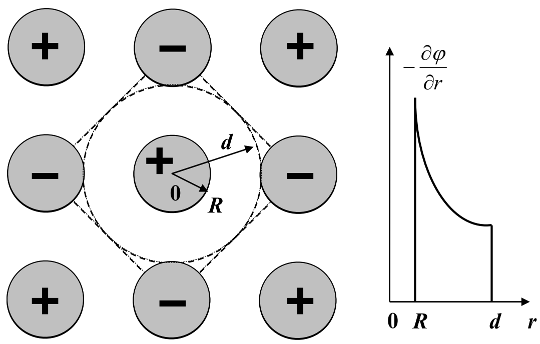

Fig. 1. Example of a flat structure formed of plasma ions. The electroneutrality condition is satisfied in the sphere of radius d.

We calculate the free energy of two-temperature, three-component plasma, and demonstrate that it has a local minimum at low electronic density, high electronic temperature and when the interparticle distance is comparable with the Debye radius. We also determine the size of the interparticle attraction existence area. We believe that such plasma can simulate the characteristics of a fireball substance.

2. Problem setting and solution

We use the scheme of calculation of free energy similar to that reported in [22]:

1. Construct the Poisson–Boltzmann equation and boundary conditions for the potential of the charged particle in three-component two-temperature plasma.
2. By means of the constructed equation, we calculate the particle potential in plasma and subtract the proper Coulomb potential of the particle. Thereby, we determine the correlation potential created by the plasma on the surface of a particle.
3. Calculate the correlation and free energy of a particle, and investigate their values at a minimum.

We believe that plasma with optimum structure, temperature and density can have a minimum of free energy and, therefore, tend to some spatial order. Thus, we consider hypothetical infinite three-dimensional plasma with ions ordered similar to arrangement of ions inside an ionic crystal. Let the plasma consist of three grades of particles: single-charged positive ions with radius R and concentration N_+; single-charged negative ions with radius R and concentration N_-; and electrons with concentration N_e. Then, the equation of electroneutrality of the plasma looks like $N_+ = N_- + N_e$. At this point, we denote the number of electrons contributing to the total quantity of negatively charged particles, $\chi = N_e/(N_e + N_-)$, where $0 < \chi \leqslant 1$, and let's also divide both members of the equation into N_+. Then the electroneutrality equation per one positive ion will be as follows:

$$\underbrace{1}_{\substack{\text{quantity}\\ \text{of positive}\\ \text{ions}}} = \underbrace{(1-\chi)}_{\substack{\text{quantity}\\ \text{of negative}\\ \text{ions}}} + \underbrace{\chi}_{\substack{\text{quantity}\\ \text{of electrons}}}. \qquad (1)$$

We define that positive and negative ions possess identical temperatures T_i. The temperature of the electrons, T_e, is considered to be different from the temperature of the ions ($T_e > T_i$).

We fix the origin of the coordinate system at the centre of any positively charged ion, as shown in Fig. 1. The electric field of the ion is partially screened by the electrons (quantity χ), located in the interionic space. Complete screening of the ion field is performed by the negative ions in the first coordination sphere of radius d (the d-sphere, $d > R$), with quantity $(1-\chi)$. Please note that the number of electrons χ, and of negative ions $(1-\chi)$, making part of screening of a field of one positive ion, are both less than one unit. It is nothing strange here, as it is only mathematical result. Actually, each electron participates in screening of the fields of several positive ions, that is the electron belongs to a concrete positive ion partially. Also, each of 5–6 (for a simple cubic lattice with small irregularities in ideality) of negative ions belonging to the first coordination sphere of a positive ion, belongs also to 5–6 coordination spheres of the neighboring positive ions. As a result, the coordination sphere of one positive ion shares $(1-\chi)$ of negative ions.

d is equal to half of the distance between nearest-neighbor like-charged ions. We consider, for simplicity, that the charge of the negative ions is evenly distributed over the d-sphere surface. (This assumption is rather rough, but it allows to make the problem of screening spherical and get simple analytical solutions keeping high-grade picture of the phenomenon.) The electric field on the surface of the d-sphere equals zero, due to electroneutrality. We now employ a spherical coordinates system. The Poisson–Boltzmann equation for potential ϕ inside the d-sphere, and the corresponding boundary conditions, appear as follows:

$$\Delta\varphi = \frac{e}{\varepsilon_0}N_e = \frac{e}{\varepsilon_0}N_0 \exp\left(\frac{e\varphi}{kT_e}\right), \qquad (2)$$

$$\left.\frac{\partial\varphi}{\partial r}\right|_{r=R} = \frac{-e}{4\pi\varepsilon_0 R^2},$$
$$\left.\frac{\partial\varphi}{\partial r}\right|_{r=d-0} = \frac{-e(1-\chi)}{4\pi\varepsilon_0 d^2} \qquad (3)$$

where $N_0 = \frac{\chi}{(4/3)\pi(d^3-R^3)}$ is the average concentration of electrons; $(4/3)\pi(d^3-R^3)$ is the volume inside the d-sphere available to the electrons; k is Boltzmann's constant; ε_0 is the electric constant; and e is the charge of the electron. Please note that the first boundary condition (3) sets the size of electric field on the surface of a positive ion. The second boundary condition (3) determines field partially screened by electrons on the inside of d-sphere surface. On the outer side of the d-sphere surface the electric field intensity snaps because of a surface negative charge $(1-\chi)e$, and equals zero. The electric field intensity equal to zero on the outer side of the d-sphere surface is guarantor of a full electroneutrality of the charge system inside d-sphere.

We suggest that the plasma is well-dispersed and the radius of ions is large, so that the plasma can be considered ideal. Then the exponent of (2) can be expanded into a series, such that $\frac{e\varphi}{kT_e} \ll 1$. As a result, we obtain the simple equation:

$$\Delta\varphi = \frac{1}{L^2}\left(\varphi + \frac{kT_e}{e}\right), \qquad (4)$$

where $L = (\frac{\varepsilon_0 kT_e}{e^2 N_0})^{0.5}$ is the plasma Debye length, calculated on the basis of the electron parameters.

The general solution of Eq. (4) in spherical coordinates looks like:

$$\varphi = \frac{C1}{r}\exp\left(-\frac{r-R}{L}\right) + \frac{C2}{r}\exp\left(\frac{r-R}{L}\right) - \frac{kT_e}{e}. \qquad (5)$$

We substitute (5) into (3), and define constants $C1$ and $C2$. Thus, we find the potential of a positively charged ion in plasma.

In the following, we calculate the correlation potential φ_+, created by the plasma on the surface of a positive ion, through subtraction of the Coulomb potential of the ion from the potential (5), $\frac{e}{4\pi\varepsilon_0 r}$: