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Probing quantum dissipative chaos using purity

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ABSTRACT

In this Letter, the purity of quantum states is applied to probe chaotic dissipative dynamics. To achieve this goal, a comparative analysis of regular and chaotic regimes of nonlinear dissipative oscillator (NDO) are performed on the base of excitation number and the purity of oscillatory states. While the chaotic regime is identified in our semiclassical approach by means of strange attractors in Poincaré section and with the Lyapunov exponent, the state in the quantum regime is treated via the Wigner function. Specifically, interesting quantum purity effects that accompany the chaotic dynamics are elucidated in this Letter for NDO system driven by either: (i) a time-modulated field, or (ii) a sequence of pulses with Gaussian time-dependent envelopes.

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1. Introduction

Nonlinear dissipative systems demonstrating chaotic behavior in their dynamics are still the subject of much attention [1–3]. The early studies of dissipative chaotic systems date back to the works [4]. The investigations of quantum chaotic systems are distinctly connected with the quantum–classical correspondence problem in general, and with environment induced decoherence and dissipation in particular. It has been recognized [5] that decoherence has rather unique properties for systems whose classical analogs display chaos. Several methods have been proposed to determine whether a quantum dissipative system exhibits chaotic behavior. At this point, we note that quite generally, chaos in classical conservative and dissipative systems with noise, has completely different properties. For example, strange attractors on Poincaré section can appear only in dissipative systems. The most successful approach that probes quantum dissipative chaos seems to be quantum tomographic methods based on the measurement of the Wigner function, which is a quantum quasi-distribution in phase-space. In this way, the connection between quantum and classical treatment of chaos can be realized by means of a comparison between strange attractors in the classical Poincaré section and the contour plots of the Wigner functions [6–8]. However, such manifestation of chaos seems to be hardly realized by experiments because Wigner function can only be obtained through data post-processing. On the other hand, alternative methods that probe quantum dissipative chaos involve considerations of entropic characteristics, analysis of statistics of excitation number [9,10], methods based on the fidelity decay [11], and Kullback–Leibler quantum divergence [12].

In this Letter, we discuss an indirect method that reveals quantum dissipative chaos based on an analysis of the purity of states. Purity is a quantity that measures the statistical characteristic of states and decoherence. It is defined through the density matrix ρ of the system as $P = \mathrm{Tr}(\rho^2)$. In consequence, the purity of any pure state is 1 and mixed states less than 1.

The goal of this Letter is to investigate the variation of the purity of quantum states in regular and chaotic systems. We shall show that purity allows us to distinguish between the ordered and chaotic quantum dissipative dynamics. In particular, we demonstrate that this program can be realized through the consideration of two schemes of nonlinear dissipative oscillator (NDO): one driven by a continuously modulated pump field, while the other under the action of a periodic sequence of identical pulses with Gaussian envelopes.

2. Purity and models of nonlinear oscillators

The purity of the quantum states, which is defined via the density matrix of the system as $\mathrm{Tr}(\rho^2)$, is connected to the linear entropy S_L in the following manner:

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$$S_L = 1 - \mathrm{Tr}(\rho^2) \quad (1)$$

and is related to the linear quantum divergence derived from the Kullback–Leibler quantum divergence [12]. Note that S_L can be obtained from the von Neumann entropy

$$S = \mathrm{Tr}(\rho \ln(\rho)) \quad (2)$$

as a lower-order approximation.

For an ensemble of mixed states the density matrix reads as

$$\rho = \sum P_{\psi_j} |\psi_j\rangle\langle\psi_j|, \quad (3)$$

where P_{ψ_j} is the probability of occurrence of state ψ_j. In this case, the purity takes the form

$$P = \mathrm{Tr}(\rho^2) = \sum_{l=0}^{\infty} P_{\psi_j}^2. \quad (4)$$

In particular, for thermal light with a photon population $P_{\psi_j} = \frac{\bar{n}^n}{(\bar{n}+1)^{(1+n)}}$, the purity can be calculated as

$$P = \frac{1}{2\bar{n}+1}, \quad (5)$$

where $\bar{n}$ is the mean number of thermal photons. From Eq. (5) it is evident that purity decreases as excitation number increases.

In this Letter, we employ quantum purity as a tool to analyse quantum dynamics of NDO which allows us to determine whether or not the system has reached the chaotic regimes. It is easy to realize that in general the purity of an ensemble of oscillatory states strongly depends on the operational regimes of NDO. More specifically, an increase in the excitation numbers would raise the number of mixing oscillatory states, leading to a decrease in purity which is apparent from Eq. (5). In addition, diffusion and decoherence of oscillatory modes are also relevant to the level of purity. Thus, in the following, we analyse the purity using the density matrix of NDO $\rho(t)$ for both the regular and chaotic regimes.

We consider a model of anharmonic oscillator driven by external field with time-modulated amplitude that is based on the following Hamiltonian in the rotating wave approximation:

$$H = \hbar\Delta a^+ a + \hbar(a^+ a)^2 + \hbar(f(t)a^+ + f(t)^* a), \quad (6)$$

where a^+ and a are the oscillatory creation and annihilation operators respectively, χ is the nonlinearity strength, and $\Delta = \omega_0 - \omega$ is the detuning between the mean frequency of the driving field and the frequency of the oscillator. In the case of a constant amplitude, i.e. $f(t) = \Omega$, this Hamiltonian describes a nonlinear oscillator driven by a monochromatic force. In this Letter, we shall consider two cases of driving force: (i) $f(t) = f_0 + f_1 \exp(\delta t)$, where δ is the frequency of modulation; and (ii)

$$f(t) = \Omega \sum e^{-(t-t_0-n\tau)^2/T^2}. \quad (7)$$

For the latter case, the time-dependent interaction term is proportional to the amplitude of the driving field Ω, and consists of Gaussian pulses with duration T separated by time intervals τ.

The evolution of the system of interest is governed by the following master equation for the reduced density matrix of the oscillatory mode in the interaction picture:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \sum_{i=1,2}\left(L_i \rho L_i^+ - \frac{1}{2}L_i^+ L_i \rho - \frac{1}{2}\rho L_i^+ L_i\right), \quad (8)$$

where $L_1 = \sqrt{(N+1)\gamma} a$ and $L_2 = \sqrt{N\gamma} a^+$ are the Lindblad operators, γ is a dissipation rate, and N denotes the mean number of quanta of a heath bath. To study the pure quantum effects, we focus below on cases of very low reservoir temperatures which, however, still ought to be larger than the characteristic temperature $\mathcal{T} \gg \mathcal{T}_{cr} = \hbar\gamma/k_B$.

It should be noted that in this approach dissipation and decoherence effects are described simultaneously by the same Lindblad terms in Eq. (8). Thus, this approach allows the control of dissipation and decoherence in NDO under time-modulated driving. Note, that quantum effects in NDO with a time-modulated driving force have been studied in the context of quantum stochastic resonance [13], quantum dissipative chaos [6–10], quantum interference assisted by a bistability [14] and generation of superposition of Fock states in the presence of decoherence due to the kicking of oscillatory dynamics [15].

In the semiclassical approach, the corresponding equation of motion for the dimensionless mean amplitude of oscillatory mode $\alpha = \mathrm{Tr}(\rho a)$ has the following form:

$$\frac{d\alpha}{dt} = -i[\Delta + \chi + 2|\alpha|^2\chi]\alpha + if(t)\Omega - \gamma\alpha. \quad (9)$$

This equation modifies the standard Duffing equation in the case of NDO with time-dependent coefficient.

These models seem experimentally feasible and can be realized in several experimental schemes. We mention nano-electromechanical systems and nano-optomechanical systems based on various nonlinear oscillators [16,17], and superconducting devices based on the nonlinearity of the Josephson junction (JJ) [18–20] that exhibits a wide variety of quantum oscillatory phenomena.

3. Purity as an indicator of chaos

In this section, we investigate the purity of quantum oscillatory states for various regimes of NDO by performing detailed comparative analysis of purity for both cases of regular and chaotic dynamics. In general, the purity of an ensemble of oscillatory states strongly depends on the level of the excitation numbers which is particularly apparent from Eq. (5). Therefore, in our comparative analysis, we consider regimes of regular and chaotic dynamics with the same levels of oscillatory excitation numbers. We shall consider two schemes of nonlinear dissipative oscillator (NDO) in this analysis: a NDO driven by a continuously modulated pump field; and a NDO under the action of a periodic sequence of identical pulses with Gaussian envelopes.

The time evolution of NDO driven by an external time-modulated force cannot be solved analytically, and hence suitable numerical methods have to be employed. Indeed, only NDO driven by monochromatic field has been solved analytically up to now in terms of the Fokker–Planck equation in the complex-P representation and in the steady-state regime by consideration of all orders of dissipation and decoherence. Applications of these results to some oscillatory models are given in Refs. [21,22].

We shall determine the excitation number and the Wigner functions of oscillatory mode numerically on the base of master equation by means of the quantum state diffusion method [23]. For the semiclassical approach, we shall calculate the Lyapunov exponent and the Poincaré section according to the framework of Eq. (9). For the Lyapunov exponent, the analysis is performed on the semiclassical time series determined from Eq. (9) according to [24], which gives the maximum Lyapunov exponent. Specifically, the definition of the Lyapunov exponent is given by $L = \frac{1}{\Delta t} \ln \frac{\|x_2(t)-x_1(t)\|}{\|x_2(t_0)-x_1(t_0)\|}$, where $x = (\mathrm{Re}(\alpha), \mathrm{Im}(\alpha), \beta)$, with β being the time variable defined through $d\beta/dt = 1$ which augments Eq. (9) to create an autonomous system. Note that x_2 and x_1 represent two trajectories that are very close together at the initial time t_0. Furthermore, $\Delta t = t - t_0$, with $t \to \infty$. For $L > 0$ the system shows chaotic dynamics. $L = 0$ corresponds to the case of conservative

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