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Skin effect modifications of the Resistive Wall Mode dynamics in tokamaks

Fabio Villone [a,*], Vladimir D. Pustovitov [b]

[a] Ass. Euratom/ENEA/CREATE, DIEI, Università di Cassino e del Lazio Meridionale, Via Di Biasio 43, 03043 Cassino, FR, Italy
[b] Institute of Tokamak Physics, National Research Centre 'Kurchatov Institute', Pl. Kurchatova 1, Moscow 123182, Russia

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ABSTRACT

We present the first evidence of the skin-effect modification of the Resistive Wall Mode (RWM) dynamics in a tokamak. The computations are performed with the CarMa code, using its unique ability of treating volumetric 3D conducting structures. The results prove that conventional thin-wall models and codes, assuming the thin equivalent wall located on the inner side of a real (thick) wall, may fail to get accurate estimates of RWM growth rates, since the inclusion of the skin effect makes the growth rates always larger than otherwise. The difference is noticeable even for the conventional slow RWMs and becomes substantial for faster modes. Some possible equivalent thin-wall modeling approaches are also discussed.

1. Introduction

The fusion plasma in toroidal devices is subject to various instabilities, with the most deleterious developing on the very fast Alfvenic time scale (typically microseconds). These put rather severe limitations on the achievable parameters and require careful optimization of the operation scenarios. Even well done, this still leaves a danger of undesirable slower instabilities. Such are, for example, so-called resistive wall modes (RWMs) [1–3]. The steady-state advanced scenario in ITER aims at high plasma pressure exceeding the RWM stability limit [2,3]. This requires suppression of the RWM instability, which can be performed with an active magnetic feedback system, as in the DIII-D tokamak [1–3]. Efficiency of such suppression depends on the quality of theoretical models behind the feedback algorithms.

RWMs are usually treated as slow modes [1–3] perfectly penetrating the vacuum vessel wall, so that the normal component of magnetic perturbations is assumed constant across the wall. In the theory of RWMs, this constitutes the widely used "thin wall" approximation [2,3]. The advanced tokamak scenarios require operation quite above the stability limit [1–3], hence the mode growth rate can be so high (though still far below the Alfvenic level) that the standard RWM theory reviewed in [2,3] may not be valid because of the skin effect [4,5]. The mode can be hence called "fast" RWM and, formally, placed between the usual "slow" RWM and much faster ideal MHD modes on Alfvenic time scale. In this range, the plasma inertia can be neglected and the wall resistivity still plays an important role in the mode dynamics, but the theory must be revised. In particular, in [4,5] it is analytically shown that the thin-wall approximation can yield a substantial underestimation of the growth rate of RWM, when the wall width and the penetration depth become comparable. Similar effect has been noticed earlier in [6] and confirmed in [7], though without much discussion of the underlying physics and separation of two regimes, slow and fast RWMs. In [4,5] the issue was clarified and the importance of thick-wall effect in present and planned tokamak experiments was strongly emphasized. In particular, the skin effect makes the current density to penetrate only partially in the wall, with a number of consequences that will be discussed in the following.

Here, we discuss the nonrotating (locked) modes with a real growth rate, using the CarMa computational tool [8,9], which has been recently developed for the analysis of RWM in presence of three-dimensional conducting structures. It has been successfully experimentally validated on the RFX-mod device [9–11] and extensively applied to ITER [12,13]. Contrary to other available three-dimensional RWM codes, like VALEN [14] or STARWALL [15], CarMa has the unique feature to combine the MHD models for the plasma with a rigorous volumetric treatment of the surrounding conductors, without resorting to the usual thin-wall approximation. This ability of CarMa allows us to present in this Letter the first numerical confirmation, with a three-dimensional RWM code, of the thick-wall effect predicted analytically in the cylindrical limit [4,5]. The calculations are performed in realistic toroidal geometry, and the results also give some indications about equivalent simplified thin-wall modeling of thick structures.

* Corresponding author. Tel.: +39 0776 2993674; fax: +39 0776 2993707.
E-mail address: villone@unicas.it (F. Villone).

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2. Formulations

In the traditional thin-wall models, the normal component of the magnetic field is assumed the same at the both sides of the wall. In the cylindrical approximation, calling $\tau_w = \mu_0 r_w d_w/\eta$ the wall time constant (r_w, d_w, η are the inner radius of the wall (wall radius in the following), thickness and resistivity, respectively), the dispersion relation for the RWM growth rate γ is:

$$\gamma \tau_w = \Gamma_m, \tag{1}$$

where $\Gamma_m = -2m(1 - B_m^{pl}/B_m)$ describes the plasma response (m is the poloidal wave number of the magnetic perturbation, B_m is its amplitude at the inner side of the wall, and B_m^{pl} is the contribution to B_m due to the plasma). The thin-wall limit corresponds to $d_w/\delta \ll 1$, where $\delta = \sqrt{\eta/(\mu_0\gamma)}$ is the skin depth. In terms of γ, this means $\gamma\tau_w \ll r_w/d_w$. This covers only a narrow range of instabilities observed in experiments. The gap between such slow RWMs and the ideal MHD modes on Alfvenic time scales is typically 3–4 orders of magnitude, which gives a vast room to the modes with larger γ. When the skin depth is only a small part of the wall thickness ($\gamma\tau_w \gg r_w/d_w$), the dispersion relation for the RWMs becomes [4,5]:

$$\gamma \tau_w = \Gamma_m d_w/\delta. \tag{2}$$

This takes into account both the reduction of the volume where the main part of the energy is dissipated and the related suppression of the field penetration through the wall. Compared to (1), this gives larger growth rates as we could expect for a thinner wall. Also, since δ is a function of γ, this means a different dependence of γ on Γ_m:

$$\gamma \tau_w = \Gamma_m^2 d_w/r_w. \tag{3}$$

Larger growth rate and its stronger dependence on Γ_m than expected from (1) indicates that the accuracy of the standard thin-wall approaches can be insufficient, when the skin depth becomes comparable with the wall thickness. Effective feedback controllers need to be designed on accurate models. Hence, reliable computational tools, able both to provide the correct growth rate scaling and to treat realistic geometries, are required.

The CarMa code answers this need. A coupling surface S is introduced between the plasma and the conducting structures. Solving the linearized single-fluid MHD equations neglecting the plasma mass, the (instantaneous) plasma response matrix to magnetic field perturbations on S is computed and is coupled to a 3D volumetric integral formulation of the eddy currents problem, which describes the conducting structures by means of a three-dimensional finite elements mesh – no thin-wall approximation is made. The final form of the model [8] is:

$$L^* \frac{dI}{dt} + RI = 0, \tag{4}$$

where I is a vector of 3D discrete currents in the finite elements mesh, the fully populated inductance matrix L^* includes the plasma response, and the sparse matrix R describes the resistance of the 3D structures. The RWM growth rate γ is computed as the unstable eigenvalue of the dynamical matrix $-(L^*)^{-1}R$ of system (4) (the meaning of multiple unstable eigenvalues is discussed in [9]). Using the same approach without plasma, only stable eigenvalues are found, the slowest of which is inversely proportional to the wall time constant τ_w. This quantity will be used for growth rate normalization, in order to compare cases referring to different conducting structures.

Table 1
Sensitivity of growth rate to mesh size.

n_t	n_p	n_w	$\gamma\ \tau_w$
50	30	4	5.90
50	30	8	5.92
50	30	2	5.84
100	30	2	5.83
50	60	2	5.90

3. The thick-wall effect

We refer to the plasma configuration described in [8], having a circular cross section with a major radius $R_0 = 2$ m and a minor radius $a = 0.4$ m, for which the $n = 1$ external kink mode is unstable (n is the toroidal mode number). With this plasma, several different circular resistive walls are considered, with the same major radius R_0 and minor radii from $r_w = 1.3a$ to $r_w = 1.5a$ (the latter denotes the inner side of the wall). We assume $d_w = r_w/10$; hence, the walls are geometrically thin. For each wall, we give a 3D finite elements discretization with $n_p = 30$ elements in the poloidal direction, $n_t = 50$ elements in the toroidal direction and $n_w = 4$ elements in the wall width. The adequacy of this choice is confirmed by the mesh sensitivity analysis of Table 1.

The CarMa code can only treat thick walls; in order to reproduce the thin-wall approximation, we consider walls with $r_w/d_w \gg 1$, representing them with only one radial finite element in the wall width. Fig. 1 shows the calculated behavior of the quantity $\Gamma_m = \gamma\tau_w$ as a function of r_w/d_w, for the considered plasma and a wall with $r_w = 1.3a$. In the explored range, a variation of Γ_m is only around 3%. In the following, we use the result for $r_w/d_w = 500$ as the thin-wall estimate.

Fig. 2 shows the growth rates for various wall configurations, both in the thin-wall approximation and using the full capacities of the CarMa code; also the thick-wall estimate (3), derived as the asymptote at $\gamma\tau_w \gg r_w/d_w = 10$, is reported. First of all, we notice that the thin-wall approximation fails to give a good estimate of the growth rate, except for $\gamma\tau_w \lessapprox 10$.

Secondly, the CarMa result is clearly capable of reproducing the expected quadratic behavior in terms of Γ_m, predicted by the thick-wall estimate (3). In any case, the CarMa result is always above the thin- and thick-wall estimates, as expected in cylindrical geometry [7].

To highlight the features of the thick-wall solutions, in Fig. 3 we show the current density pattern corresponding to the unstable eigenvector and the radial behavior of the toroidal component J_ϕ in the wall, for a case with $\Gamma_m = 23.6$. The current density has a dominant $n = 1$, low-m structure. A substantial attenuation of J_ϕ across the wall is precisely the effect completely neglected in the thin-wall approximation. The radial decay of the poloidal and much smaller radial component is similar.

This thick-wall effect is expected to become significant when the skin depth δ is comparable to the wall width [4,5]. To quantify this expectation, we computed the quantity $\gamma\tau_w/\Gamma_m$ as a function of the ratio d_w/δ (Fig. 4). We refer here to the configuration with $r_w = 1.5a$, considering various walls of increasing thickness. When the penetration depth δ is much greater than the wall width d_w, we recover the thin-wall approximation, so that $\gamma\tau_w \to \Gamma_m$. Conversely, when $d_w \gg \delta$, any increase of the wall thickness does not appreciably modify the growth rate γ, since the skin effect reduces the induced currents in the additional layers of the conducting material. Consequently, in this limit the quantity $\gamma\tau_w$ increases with d_w as τ_w. When $d_w \simeq \delta$ (a situation in-between the validity range of the existing analytical models [4,5]) the error of the thin-wall approximation can exceed 50%. This proves the