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A probabilistic CNOT gate for coherent state qubits

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ABSTRACT

We propose a scheme for implementing a probabilistic controlled-NOT (CNOT) gate for coherent state qubits using only linear optics and a particular four-mode state. The proposed optical setup works, as a CNOT gate, near-faithful when $|\alpha|^2 \geqslant 25$ and independent of the input state. The key element for realizing the proposed CNOT scheme is the entangled four-mode state.

1. Introduction

After Knill et al. [1] showed that linear optics alone would suffice to implement efficient quantum computing, quantum optics, that had proved to be a fertile field for experimental tests of quantum science, brought a great perspective to quantum information processing (QIP).

In [1] efficient quantum computation is achieved using single photon sources and single photon detectors, but the alternate idea of encoding quantum information on continuous variables [2] has lead to a number of proposals for realizing multi-photon [3–7] and hybrid (coherent states and single photon) [8] quantum computations. The hybrid scheme proposed in [8] is, actually, more efficient than pure linear optical and pure coherent state quantum computers.

The main drawback of proposals [3–5] is that "hard", non-linear interactions are required in-line of the computation, and these would be difficult to implement in practice.

The elegant scheme proposed in [6] requires only relatively simple linear optical networks and photon counting, but, unfortunately, the amplitude of the required superpositions of coherent states is prohibitively large. On the other hand, the scheme proposed in [7], that was built on the idea found in [6], requires only "easy", linear in-line interactions, since all the hard interactions are only required for off-line production of resource states, and is based on much smaller superposition states.

The universal set of gates presented in [7] is composed by a phase rotation gate, a superposition gate (that implements a rotation of $\pi/2$ about X) and a two-qubit controlled phase gate. If a CNOT gate using coherent states is proposed, the universal set of gates for [7] can be simplified, since any quantum circuit can be built using single qubit gates and CNOTs. Our goal here is to propose a scheme for implementing probabilistically a CNOT gate for coherent state encoded qubits using an entangled four-mode state, beam splitters and photon number counters.

Several proposals and experimental implementations of a CNOT gate for single photon qubits have been done in the last years [9,10]. Pittman et al. describe in [9] a quantum parity check and a quantum encoder and show how they may be combined to implement a CNOT gate using polarizing beam splitters and polarization single photon qubits. The experimental demonstration of this gate can be found in [11]. It is described in [12] the operation and tolerances of a nondeterministic, coincidence basis, quantum CNOT gate for photonic qubits. The gate is constructed using linear optical elements and requires only a two-photon source for its demonstration. Its success probability is 1/9.

An unambiguous experimental demonstration and comprehensive characterization of quantum CNOT operation in an optical system using four entangled Bell states as a function of only the input qubits' logical values, for a single operating condition of the gate, is found in [13]. The gate is probabilistic, but with the addition of linear optical quantum non-demolition measurements, it is equivalent to the CNOT gate required for scalable all-optical quantum computation.

In [14] it is reported an experimental demonstration of teleportation of a CNOT gate assisted with linear optical manipulations, photon entanglement produced from parametric down-conversion, and postselection from the coincidence measurements. The average fidelity for the teleported gate is 0.84. Zhao et al. detail in [10] a proof-of-principle experimental demonstration of a nondestructive

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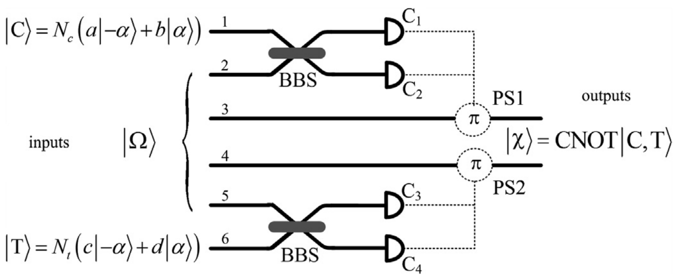

Fig. 1. Optical setup for performing the CNOT gate for coherent state qubits probabilistically.

CNOT gate for two independent photons using only linear optical elements in conjunction with single-photon sources and conditional dynamics.

All the examples given above are probabilistic gates. A deterministic CNOT is still not available due to the need of non-linear operation [15,16]. Here we present a proposal for implementing probabilistically a CNOT gate inspired by the scheme presented in [9].

This Letter is outlined as follows: in Section 2 we present the optical setup for a probabilistic CNOT gate for coherent state encoded qubits; in Section 3 brings the analysis of success and fidelity of the proposed CNOT gate; and, at last, we make our concluding remarks in Section 4.

2. Optical setup for a probabilistic CNOT gate

We intend to perform a CNOT gate between the qubits $|C\rangle = a|0\rangle + b|1\rangle$ and $|T\rangle = c|0\rangle + d|1\rangle$, where $|C\rangle$ and $|T\rangle$ are the control and the target qubits, respectively. In a coherent state quantum computer (CSQC), the qubit is encoded as $|0\rangle_L = |-\alpha\rangle$ and $|1\rangle_L = |\alpha\rangle$ where α is assumed to be real. In this case, we have $|\langle 0|1\rangle|^2 = |\langle -\alpha|\alpha\rangle|^2 = \exp(-4|\alpha|^2)$, which ensures the orthogonality if $\alpha \geqslant 2$ [3–6]. Thus, the states $|C\rangle$ and $|T\rangle$ for coherent state qubits are: $|C\rangle = N_c(a|-\alpha\rangle + b|\alpha\rangle)$ and $|T\rangle = N_t(c|-\alpha\rangle + d|\alpha\rangle)$, where $N_c = [1 + 2\cdot\Re(a^*b)\exp(-2|\alpha|^2)]^{-1/2}$ and $N_t = [1 + 2\cdot\Re(c^*d)\exp(-2|\alpha|^2)]^{-1/2}$ are normalization constants.

Schematic of the optical setup for our proposed CNOT gate is showed in Fig. 1. The state $|\Omega\rangle$ in Fig. 1 is an four-mode state given by $|\Omega\rangle = N_\Omega(|-\alpha,-\alpha,-\alpha,-\alpha\rangle + |-\alpha,-\alpha,\alpha,\alpha\rangle + |\alpha,\alpha,\alpha,-\alpha\rangle + |\alpha,\alpha,-\alpha,\alpha\rangle)$, where the normalization constant is $N_\Omega = 4[1+\exp(-4|\alpha|^2) + 2\cdot\exp(-6|\alpha|^2)]^{-1/2}$. This state can be generated by the quantum circuit shown in Fig. 2 and can be implemented nondeterministically from the optical scheme proposed in [26]. The success probability of this scheme is 1/4.

In Fig. 1, BS, PS and C are beam splitters, phase shifters and photon counters, respectively. The set of beam splitters and photon counters are used to perform Bell-state measurements [17,18]. The unitary operator of a lossless balanced beam splitter is $\hat{B} = \exp[\pi(\hat{a}_1\hat{a}_2^\dagger + \hat{a}_1^\dagger\hat{a}_2)/4]$. If we send two coherent states $|\alpha\rangle_1$ and $|\beta\rangle_2$ through the BS, the total state at the output is given by:

$$\hat{B}|\alpha,\beta\rangle_{1,2} = \left|(\alpha-\beta)/\sqrt{2}, (\alpha+\beta)/\sqrt{2}\right\rangle_{1,2}. \quad (1)$$

The PS, by its turn, adds a phase θ to the signal passing through it. Its unitary operator is $\hat{U}(\theta) = \exp(j\theta\hat{a}^\dagger\hat{a})$, such that:

$$\hat{U}|\alpha\rangle = \left|e^{j\theta}\alpha\right\rangle. \quad (2)$$

If $\theta = \pi$, the PS is a NOT or X gate for a CSQC because if the light entering the PS is a coherent state $|\alpha\rangle(|-\alpha\rangle)$, the output state will be $|-\alpha\rangle(|\alpha\rangle)$.

Still referring to Fig. 1, mode 1 is the control qubit $|C\rangle$, mode 6 is the target qubit $|T\rangle$ and modes 2 to 5 correspond to the auxiliary resource state $|\Omega\rangle$. Before the photon counters, the state $|\psi\rangle$, resulting from the evolution of the input state $|C\rangle_1 \otimes |\Omega\rangle_{2-5} \otimes |T\rangle_6$ through the optical setup, is given by:

$$\begin{aligned}
|\psi\rangle = N\big[& ac\big(|0,-\sqrt{2}\alpha,-\alpha,-\alpha,0,-\sqrt{2}\alpha\rangle \\
& + |0,-\sqrt{2}\alpha-\alpha,\alpha,\sqrt{2}\alpha,0\rangle \\
& + |-\sqrt{2}\alpha,0,\alpha,\alpha,0,-\sqrt{2}\alpha\rangle \\
& + |-\sqrt{2}\alpha,0,\alpha,-\alpha,\sqrt{2}\alpha,0\rangle\big) \\
& + ad\big(|0,-\sqrt{2}\alpha,-\alpha,-\alpha,-\sqrt{2}\alpha,0\rangle \\
& + |0,-\sqrt{2}\alpha-\alpha,\alpha,0,\sqrt{2}\alpha\rangle \\
& + |-\sqrt{2}\alpha,0,\alpha,\alpha,-\sqrt{2}\alpha,0\rangle \\
& + |-\sqrt{2}\alpha,0,\alpha,-\alpha,0,\sqrt{2}\alpha\rangle\big) \\
& + bc\big(|\sqrt{2}\alpha,0,-\alpha,-\alpha,0,-\sqrt{2}\alpha\rangle \\
& + |\sqrt{2}\alpha,0,-\alpha,\alpha,\sqrt{2}\alpha,0\rangle \\
& + |0,\sqrt{2}\alpha,\alpha,\alpha,0,-\sqrt{2}\alpha\rangle + |0,\sqrt{2}\alpha,\alpha,-\alpha,\sqrt{2}\alpha,0\rangle\big) \\
& + bd\big(|\sqrt{2}\alpha,0,-\alpha,-\alpha,-\sqrt{2}\alpha,0\rangle \\
& + |\sqrt{2}\alpha,0,-\alpha,\alpha,0,\sqrt{2}\alpha\rangle \\
& + |0,\sqrt{2}\alpha,\alpha,\alpha,-\sqrt{2}\alpha,0\rangle \\
& + |0,\sqrt{2}\alpha,\alpha,-\alpha,0,\sqrt{2}\alpha\rangle\big)\big], \quad (3)
\end{aligned}$$

where $N = N_c \cdot N_\Omega \cdot N_t$. When the photon counter C_x registers n_x photons, we obtain one of the following states on modes 3 and 4:

$$\begin{aligned}
|\chi\rangle_{3,4} &= {}_{1,2,5,6}\langle 0,n_2,0,n_4|\psi\rangle_{1-6} \\
&\simeq ac(-1)^{n_2+n_4}|-\alpha,-\alpha\rangle + ad(-1)^{n_2}|-\alpha,\alpha\rangle \\
&\quad + bc(-1)^{n_4}|\alpha,\alpha\rangle + bd|\alpha,-\alpha\rangle, \quad (4)
\end{aligned}$$

$$\begin{aligned}
|\chi\rangle &= \langle n_1,0,n_3,0|\psi\rangle \\
&\simeq ac(-1)^{n_1}|\alpha,-\alpha\rangle + ad(-1)^{n_1+n_3}|\alpha,\alpha\rangle \\
&\quad + bc|-\alpha,\alpha\rangle + bd(-1)^{n_3}|-\alpha,-\alpha\rangle, \quad (5)
\end{aligned}$$

$$\begin{aligned}
|\chi\rangle &= \langle 0,n_2,n_3,0|\psi\rangle \\
&\simeq ac(-1)^{n_2}|-\alpha,\alpha\rangle + ad(-1)^{n_2+n_3}|-\alpha,-\alpha\rangle \\
&\quad + bc|\alpha,-\alpha\rangle + bd(-1)^{n_3}|\alpha,\alpha\rangle, \quad (6)
\end{aligned}$$

$$\begin{aligned}
|\chi\rangle &= \langle n_1,0,0,n_4|\psi\rangle \\
&\simeq ac(-1)^{n_1+n_4}|\alpha,\alpha\rangle + ad(-1)^{n_1}|\alpha,-\alpha\rangle \\
&\quad + bc(-1)^{n_4}|-\alpha,-\alpha\rangle + bd|-\alpha,\alpha\rangle. \quad (7)
\end{aligned}$$

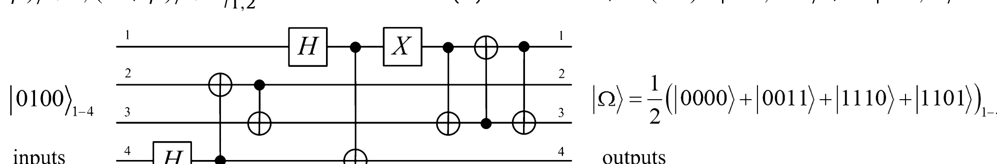

Fig. 2. Circuit to generate a four-partite entangled state $|\Omega\rangle$ for single photon qubits.