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Protecting entanglement of atoms stored in a common nonperfect cavity without measurements

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ABSTRACT

We study entanglement dynamics of two and three atoms stored in a common nonperfect cavity together with some other nonentangled atoms. It is guessed at first thought that the presence of nonentangled atoms would favor the decoherence process of the interested entangled atoms. We show, on the contrary, that it is not so. Namely, as results of a rigorous nonperturbative analysis, disentanglement rate of the interested atoms decreases with the increase of the number of nonentangled atoms. If the number of nonentangled atoms is sufficiently large, the entanglement of interested atoms could be protected efficiently.

1. Introduction

The 'spooky-action-at-distance' feature of special quantum states was disliked by Einstein who, however, exploited it in an attempt to convince the incompleteness of quantum mechanics in 1935 [1]. Later in the same year, in a gedanken experiment aiming at a possible demonstration of how the microscopic and macroscopic worlds can directly be coupled to each other [2], Schrödinger coined that 'spooky' feature entanglement (verschränkung in German) and the states possessing it entangled states. Nowadays, it is ubiquitous that entanglement offers a vital shared resource allowing to perform various quantum network protocols only by means of local operations and classical communication (see, e.g., [3]). After production and before distribution to remote authorized parties, entangled states are often stored for some time in a register. But during the storage process entangled states are degrading and tend to be separable for a sufficiently long time. Therefore, protecting entanglement for a later use is of paramount importance.

In this Letter we are interested in bipartite and tripartite entangled states of two-level atoms (served as qubits) stored in a common nonperfect cavity together with some $\mathcal{N}$ additional atoms which are not entangled with the interested atoms as well as with each other. At first thought one might guess that the presence of such additional atoms would cause a negative affect on the quality of the atomic entangled states when all the atoms evolve due to interaction with the cavity modes. We shall develop exact theory of the dynamics of a general class of multiatom states and then base on it to explicitly show that the entanglement degradation of EPR-type and W-type states turns out to slow down as the number of the additional atoms increases. Theoretically, the entanglement quality could be invariant in the large-$\mathcal{N}$ limit.

2. EPR-type entanglement

In Ref. [4] the authors analyzed the exact entanglement dynamics of two two-level atoms initially prepared in an EPR-type state

$$\left|epr(0)\right\rangle_{12} = \left(a_1(0)|10\rangle + a_2(0)|01\rangle\right)_{12}, \tag{1}$$

with $|a_1(0)|^2 + |a_2(0)|^2 = 1$ (which reduces to an EPR state [1] when $|a_1(0)| = |a_2(0)|$) and $|0\rangle$ ($|1\rangle$) the atom ground (excited) state, inside a common lossy (i.e., nonperfect) cavity at zero-temperature with the Lorentzian spectral density

$$J(\omega) = \frac{R^2}{\pi}\frac{\Gamma}{(\omega-\omega_c)^2+\Gamma^2}, \tag{2}$$

where ω_c is the frequency of the mode supported by the cavity, Γ^2 describes the leakage probability of the cavity mode through the nonideal walls and R measures the atom–cavity coupling strength. The result obtained is that the entanglement degree measured by concurrence [6] decreases quickly with time in both the bad (i.e., weak coupling or Markovian regime) and good (i.e., strong coupling or non-Markovian regime) cavity limits. To fight against the entanglement deterioration the authors proposed a method by using the quantum Zeno effect, that requires a series of frequent specific nonselective measurements on the collective atomic system. Alternatively, atomic entanglement can also be protected by

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weak measurements combined with quantum measurement reversals [5]. Here, we propose another method that does not need any measurements. Instead, we add some $\mathcal{N}$ auxiliary atoms in the ground state and let all the atoms evolve inside the cavity. That is, at $t=0$ the total atom–cavity system state is

$$\left|\psi(0)\right\rangle_s = \left|epr(0)\right\rangle_{12}|00\ldots0\rangle_{34\ldots N}|\bar{\mathbf{0}}\rangle_c, \tag{3}$$

where $N=\mathcal{N}+2$ and $|\bar{\mathbf{0}}\rangle_c = \bigotimes_j |0_j\rangle_c$ with $|0_j\rangle_c$ the cavity state containing zero photon in mode j.

To see how the total system state (3) evolves in time, let us study a more general problem as follows. Consider a nonperfect cavity at zero-temperature with the spectral density (2) that contains in it an N-atom ($N \geqslant 3$) state of the form

$$\left|\Phi(0)\right\rangle_{12\ldots N} = \big(a_1(0)|10\ldots0\rangle + a_2(0)|01\ldots0\rangle + \cdots + a_N(0)|00\ldots1\rangle\big)_{12\ldots N}, \tag{4}$$

with $\sum_{n=1}^N |a_n(0)|^2 = 1$. Extending the case of $N=2$ in Ref. [4] to an arbitrary N, the total atom–cavity system Hamiltonian is ($\hbar = 1$)

$$\hat{H} = \hat{H}_0 + \hat{H}_{int}, \tag{5}$$

with

$$\hat{H}_0 = \sum_{n=1}^N \Omega\hat{\sigma}_n^+\hat{\sigma}_n + \sum_j \omega_j \hat{a}_j^+\hat{a}_j \tag{6}$$

and

$$\hat{H}_{int} = \sum_{n=1}^N \sum_j \alpha_n\big(g_j\hat{\sigma}_n^+\hat{a}_j + g_j^*\hat{\sigma}_n\hat{a}_j^+\big). \tag{7}$$

In the above equations $\hat{\sigma}_n = |0\rangle_n\langle 1|$, Ω is the atomic transition frequency, while ω_j and $\hat{a}_j$ ($\hat{a}_j^+$) are the frequency and the annihilation (creation) operator of the cavity mode-j photon. The interaction between the nth atom and the mode-j photon is described by $\alpha_n g_j$, with α_n a real positive dimensionless constant whose value depends on the value of the cavity field at the atom position and can be effectively adjusted, say, by tuning the atomic transition thanks to dc Stark effect [4]. In what follows, as in Ref. [4], $\alpha = \sqrt{\sum_{n=1}^N \alpha_n^2}$ and $r_n = \alpha_n/\alpha$ (so that $\sum_{n=1}^N r_n^2 = 1$) are introduced for convenience.

The total system state at $t=0$ is then

$$\left|\Psi(0)\right\rangle_s = \left|\Phi(0)\right\rangle_{12\ldots N}|\bar{\mathbf{0}}\rangle_c. \tag{8}$$

Since $\sum_j \hat{a}_j^+\hat{a}_j + \sum_{n=1}^N \hat{\sigma}_n^+\hat{\sigma}_n$ commutes with $\hat{H}$, at time $t>0$ the state (8) evolves into

$$\left|\Psi(t)\right\rangle_s = e^{-i\Omega t}\left|\Phi(t)\right\rangle_{12\ldots N}|\bar{\mathbf{0}}\rangle_c + \sum_j b_j(t)e^{-i\omega_j t}|00\ldots0\rangle_{12\ldots N}|1_j\rangle_c, \tag{9}$$

with $|1_j\rangle_c$ the cavity state containing only one photon in mode j. The time-dependent coefficients $a_n(t)$ and $b_j(t)$ can be derived from the equation of motion governed by $\hat{H}$:

$$i\frac{da_n(t)}{dt} = \alpha_n \sum_j g_j e^{-i(\omega_j-\Omega)t} b_j(t), \tag{10}$$

$$i\frac{db_j(t)}{dt} = g_j^* e^{i(\omega_j-\Omega)t}\sum_{n=1}^N \alpha_n a_n(t). \tag{11}$$

Solving Eq. (11) for $b_j(t)$ with the initial condition $b_j(0)=0$ then substituting it into Eq. (10) yields

$$\frac{da_n(t)}{dt} = -\int_0^t \int d\omega\, J(\omega)e^{-i(\omega-\Omega)(t-t')}\alpha_n \sum_{m=1}^N \alpha_m a_m(t')\,dt'. \tag{12}$$

By Laplace transforming both sides of Eq. (12) we obtain an algebraic closed set of equations for $\{\tilde{a}_n(z) = \mathcal{L}[a_n(t)]\}$. Solving this equation set and inverse Laplace transforming the solutions gives

$$a_n(t) = a_n(0) - r_n F(t)\sum_{m=1}^N r_m a_m(0) \tag{13}$$

where

$$F(t) = 1 - e^{-(\Gamma+i\Delta)t/2}\Bigg[\cosh\bigg(\frac{t}{2}\sqrt{(\Gamma+i\Delta)^2 - 4\alpha^2R^2}\bigg) + \frac{\Gamma+i\Delta}{\sqrt{(\Gamma+i\Delta)^2-4\alpha^2R^2}} \times \sinh\bigg(\frac{t}{2}\sqrt{(\Gamma+i\Delta)^2-4\alpha^2R^2}\bigg)\Bigg], \tag{14}$$

with $\Delta = \omega_c - \Omega$. Defining the single-photon collective normalized state of the cavity field as

$$|\bar{\mathbf{1}}\rangle_c = \frac{e^{i\omega_c t}}{b(t)}\sum_j b_j(t)e^{-i\omega_j t}|1_j\rangle_c, \tag{15}$$

with

$$b(t) = \sqrt{1 - \sum_{n=1}^N \left|a_n(t)\right|^2}, \tag{16}$$

we obtain the explicit expression for $|\Psi(t)\rangle_S$:

$$\left|\Psi(t)\right\rangle_s = e^{-i\Omega t}\big[a_1(t)|10\ldots0\rangle + a_2(t)|01\ldots0\rangle + \cdots + a_N(t)|00\ldots1\rangle\big]_{12\ldots N}|\bar{\mathbf{0}}\rangle_c + e^{-i\omega_c t}b(t)|00\ldots0\rangle_{12\ldots N}|\bar{\mathbf{1}}\rangle_c, \tag{17}$$

with $a_n(t)$ and $b(t)$ determined by Eqs. (13) and (16), respectively.

Returning now to the atoms' state of interest, i.e., state $|epr(0)\rangle_{12}|00\ldots0\rangle_{34\ldots N}$ with $|epr(0)\rangle_{12}$ given by Eq. (1). It can be verified that state $|epr(0)\rangle_{12}|00\ldots0\rangle_{34\ldots N}$ is a particular case of the states $|\Phi(0)\rangle_{12\ldots N}$ in Eq. (4) with $a_3(0)=a_4(0)=\cdots=a_N(0)=0$. So the above analytical results for the general case apply to the dynamics of EPR-type endamagement which we are interested in in this section. Concretely, according to the general results derived above, at any $t>0$ state (3) evolves into state (17) with

$$a_n(t) = a_n(0) - r_nF(t)\big(r_1a_1(0) + r_2a_2(0)\big). \tag{18}$$

The atomic state $\rho^{12\ldots N}(t)$ is obtained by tracing out over the field modes:

$$\rho^{12\ldots N}(t) = \mathrm{Tr}_c\left|\Psi(t)\right\rangle_s\left\langle\Psi(t)\right| = \left|\Phi(t)\right\rangle_{12\ldots N}\left\langle\Phi(t)\right| + \left|b(t)\right|^2|00\ldots0\rangle_{12\ldots N}\langle 00\ldots0|. \tag{19}$$

The state of the two interested atoms 1 and 2 is then described by the reduced density matrix $\rho^{12}(t) = \mathrm{Tr}_{34\ldots N}\,\rho^{12\ldots N}(t)$. After the necessary tracing procedure we arrive at

$$\rho^{12}(t) = \Bigg(\left|b(t)\right|^2 + \sum_{m=3}^N\left|a_m(t)\right|^2\Bigg)|00\rangle_{12}\langle 00| + \big(a_1(t)|10\rangle + a_2(t)|01\rangle\big)_{12}\big(a_1^*(t)\langle 10| + a_2^*(t)\langle 01|\big). \tag{20}$$