Physics Letters A 377 (2013) 2542–2545

Contents lists available at SciVerse ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Detrended fluctuation analysis and Kolmogorov–Sinai entropy of electroencephalogram signals

Jung Ho Lim [a], Eun Joo Khang [a], Tae Hyun Lee [a], In Hye Kim [a], Seong Eun Maeng [b], Jae Woo Lee [b,*]

[a] Gyeonggi Science High School, Soo Won, Gyeonggido 440-800, Republic of Korea
[b] Department of Physics, Inha University, Incheon 402-751, Republic of Korea

ARTICLE INFO

Article history:
Received 18 April 2013
Received in revised form 1 July 2013
Accepted 18 July 2013
Available online 25 July 2013
Communicated by A.R. Bishop

Keywords:
Electroencephalogram
Detrended fluctuation analysis
Hurst exponent
Entropy

ABSTRACT

We measured the electroencephalogram (EEG) of young students in the relaxed state and in the state of the mathematical activities. We applied the detrended fluctuation analysis and Kolmogorov–Sinai entropy (KSE) in the EEG signals. We found that the detrended fluctuation functions follow a power law with Hurst exponents larger than 1/2. The Hurst exponents enhanced at all EEG channels in the state of mathematical activities. The KSE in the relaxed state is larger than those in the state of the mathematical activities. These indicate that the entropy is enhanced in the disorder state of the brain.

1. Introduction

The brain activities have been measured by the electroencephalogram (EEG) signals. The EEG time series provide many information of the human brain dynamics. The properties of the EEG signals have been analyzed by a lot of methods such as information theory, nonlinear dynamic methods, fractal theory, methods of the complex system, etc. [1–9]. Because the EEG signals are nonlinear time series, many nonlinear dynamic methods are successfully applied to measure the physical properties of the EEG signals. They include the entropy measure [10,11], the Lyapunov exponent [12,13], fractal dimension [14,15], the Hurst exponents [16, 17], and the multiscaling properties [18,19]. The nonlinear dynamic methods gave the ways to distinguish between the patient with the epileptic seizures and the healthy person by the EEG signals [20–22]. There were many works for the effects of the electromagnetic (EM) field on the human brain [23–29]. The increasing use of the electronic equipments has raised the questions of whether emitted EM fields influence human brain function [23–27]. The effects of exposure to the EM field on human brain activity have been examined by various methods for the electroencephalogram (EEG) time series in both wake and sleep state [28,29].

The method of the DFA measures the persistent or the correlated properties of the time series. The DFA method has been successfully applied to measure the long-range correlation in the time series such as a foreign exchange rate, a return of a stock market index, a time series of the heart beat and physiological data, etc. [6–8]. The Kolmogorov–Sinai (KS) entropy characterize the information or the disorder of the nonlinear time series [6].

In this work we have measured the EEG signals of young high school students in the relaxed state and in the state of the mathematical activities. We applied the detrended fluctuation analysis (DFA) and Kolmogorov–Sinai entropy (KSE) to the EEG time series. We obtained the Hurst exponents by the detrended fluctuation function and the KS entropy in two experimental conditions. The DFA and KSE are good methods to measure the activities of the brain in the EEG signals. In Section 2 we introduce the measuring method of the EEG signals. In Section 3 we reviewed the DFA method and KSE method. In Section 4 we gave the experimental results and discussions. The final conclusion gave in Section 5.

2. Measurement of EEG

We have measured the 8 channel EEG signals for 25 young and health high school students of age 17. We use LXE 3208 (AXTHA) equipment to measure the EEG signals. We attached eight electrodes on the brain following the international standard system. The reference electrodes are attached on the two ears. We select typical eight channels: Fp1, Fp2 on the prefrontal lobe, F3, F4 on the frontal lobe, T5, T6 on the temporal lobe, and P3, P4 on the

* Corresponding author. Tel.: +82 32 860 7660; fax: +82 32 872 7672.
E-mail address: jaewlee@inha.ac.kr (J.W. Lee).

0375-9601/$ – see front matter
http://dx.doi.org/10.1016/j.physleta.2013.07.034

parietal lobe. We measured the EEG signals when the brain is in the relaxed state and in the state of the mathematical activities. In the relaxed state the students stay doing nothing and watch a black circle in front of them which is away about 5 m. Watching the black circle reduces the abrupt occurrences of the noisy signals in the EEG signals in the relaxed state. In the state of the mathematical activities the students add consecutively two numbers consisting of the two or three digit integers. The mathematical problems displayed on the screen of the laptop computers sequentially. During they solve the problems we measured the EEG time series. We expect the changes of the EEG signals in the brain activities as one solves the mathematical problems. The EEG signals were recorded per every four milliseconds. We measured the EEG time series during three minutes. The total number of recorded data at every channel is about 45 000.

3. Detrended fluctuation analysis and KS entropy

Consider an EEG signals $x(1), x(2), \dots, x(N)$. The detrended fluctuation analysis (DFA) characterizes the long-term correlations in a time series [5,9]. We divide the time series of length N into $[N/l]$ nonoverlapping segments, each containing l data points. Then, we remove the local trend in each segment. The local trend is obtained by a least-squares fit in that segment. Finally we calculate the detrended time series, $y_l(i) = x_l(i) - f_l(i)$, where $f_l(i)$ is a trend function. Here, we use a linear trend function. If we use higher polynomial function as a detrend function, we obtained the same results in the method of the DFA. The detrended fluctuation functions is defined by

$$F_d(l) = \left\langle \sum_{i=1}^{l} y_l^2(i) \right\rangle \sim l^{2H}, \tag{1}$$

where the average is taken over the all segments. The exponent H is called the Hurst exponent. If the time series is random and uncorrelated, the Hurst exponent is $H = 1/2$. However, the Hurst exponent deviates from $1/2$ when the time series is correlated. If $H > 1/2$, the system shows the persistent behaviors. If $H < 1/2$, the system reveals the antipersistent behaviors.

The rate of creation of information is characterized by the Kolmogorov–Sinai (KS) entropy in a nonlinear dynamic system [5]. In the chaotic system the sum of all the positive Lyapunov exponents is equivalent to the KS entropy. We regenerate the time series in an embedding dimension [5]. Then we calculate the correlation function

$$C(d, \epsilon) = \frac{1}{M^2} \sum_{i=1}^{M} \Theta\left(\epsilon - |x_i - x_j|\right), \tag{2}$$

where Θ is the Heaviside function, x_i and x_j are reconstructed vectors in the embedding dimension d, M is the number of points in the reconstructed phase space, and ϵ is a prescribed small distance. The correlation function is given by

$$C(d, \epsilon) \sim \epsilon^{D_2} \exp(-dK_2), \tag{3}$$

where we set the delay time and sampling time as a unit value. D_2 is the correlation dimension, and K_2 is the KS entropy. We obtained the KS entropy from the slopes when we plot the correlation function as $\ln C = -dK_2 + a$ where a is a constant.

4. Results and discussions

Fig. 1 represented the parts of the EEG signals recorded at the channel Fp1 of the prefrontal lobe for a student in the relaxed state in Fig. 1(a) and in the state of mathematical activities in Fig. 1(b). In the state of the mathematical activities the students are solving some mathematical problems. They add consecutively two numbers consisting of two or three digit numbers. The amplitudes of the EEG signals in the state of mathematical activities are larger than those in the relaxed state. The behaviors of the two time series in Fig. 1 are very similar each other. The EEG signals have been recorded during three minutes at each channel for each student. The total number of data at each channel corresponds to 45 000 points. Therefore, we need to distinguish two time series by any physical quantities. Many methods in the nonlinear dynamics were applied to analyze the EEG time signals [1–5]. The detrended fluctuation method and the KS entropy are good candidates to observe the nonlinearity of the time series. Shao et al. showed that the DFA and CDMA (centred detrended moving average) were the best methods to estimate the Hurst exponents and were insensitive to the scaling range [30]. We applied the DFA method and the KS entropy to the EEG signal and observed the different exponents under the experimental conditions.

We consider the detrended fluctuation function varying the size of the windows in the EEG signals. Each window contains l data points. The local trend of the each window is given by

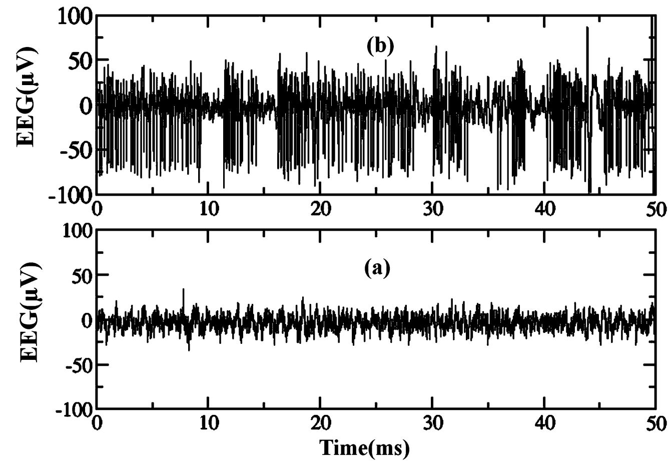

Fig. 1. The parts of the EEG signals measured on the prefrontal lobe (Fp1 channel) of a subject as a function of time (a) in a relaxed state, and (b) in a state of the mathematical activities.

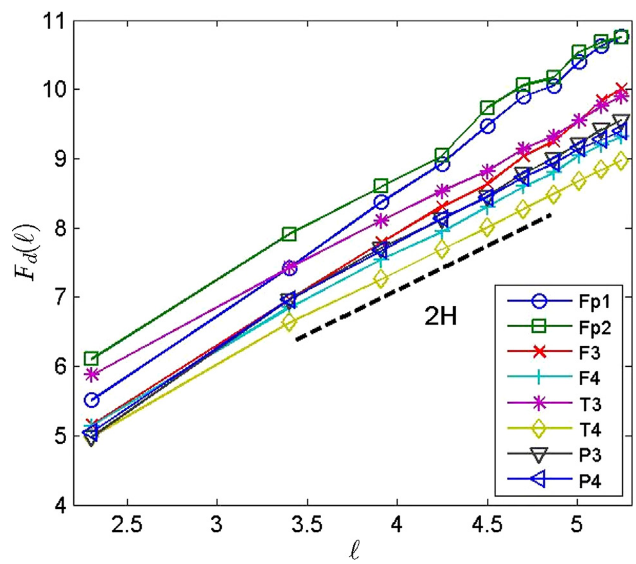

Fig. 2. The log–log plot of the average detrended fluctuation function against the size of the window in a relaxed state on a subject. We observed the obvious scaling region in the plot. The slopes correspond to the $2H$ where H is the Hurst exponent.