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Modulational instability of ultra-intense linearly polarized laser pulse in electron–positron plasmas

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ABSTRACT

Based on the wave equation of ultra-intense linearly polarized laser pulse propagating in electron–positron plasmas, the modulational instability is investigated. The nonlinear dispersion relation and the growth rate of instability are derived. The effects of plasmas number density, temperature, and laser intensity on the growth rate are analyzed. Results show that in an electron–positron plasma with certain background density, the intensity of the modulation instability is mainly determined by the competition between the nonlinearity in the interaction and the relativistic light ponderomotive driven density responses.

1. Introduction

Electron–positron (e–p) plasmas are ubiquitous in the early Universe and relevant in many environments, such as accretion disks, pulsar magnetospheres, neutron stars, cosmic solar flares, black holes, and so on [1–4]. Whereas, due to the special environments that e–p plasmas can exist, the plasma physics of such relativistic e–p pairs is poorly understood [3]. However, fast progressing ultra-intensity laser technology offers the perspective to create relativistic plasmas composed of electrons and positrons under laboratory conditions. Researchers have shown that a 10 ps, 10^{20} W/cm^2 laser hitting solid density gold foils on both sides can in principle produce peak pair density of the order of 10^{-3} of the target electron density [4]. An alternative approach to achieve a clean pair-dominated plasma is to let the above plasma freely expand after laser turn off. Then, since the pairs are thousands of times less massive than the ions, they will expand much faster than the ions. After many e-foldings of expansion, the leading rarefaction front will be pure pairs, leaving the ions and background electrons behind. Using ultra-intense laser pulse and high Z targets interaction can produce e–p plasma is undoubtedly something worthy to be happy, because it means that people can simulate cosmic environment in the laboratory, thus opening a new way to explore the mysteries of the universe [5].

The study of the nonlinear interaction of ultra-intense laser pulse with e–p plasmas is very important for people to understand some physical processes that occurred in e–p plasmas and it has received much attention in the last decade [6–9]. The properties of e–p plasmas is different from electron–ion (e–i) plasmas. In the e–i plasmas, due to the mass of ion is much larger than the mass of electron, the ion is considered as motionless in many situations. So, under the action of ultra-intense shot laser pulse, a strong electrostatic separation field can be generated and the perturbation of plasmas density is mainly determined by the competition of electrostatic force and ponderomotive force. But in e–p plasmas, due to the equal mass of electron and positron, the motion of them are symmetric and the perturbation of plasmas density is mainly determined by the competition of thermal pressure and ponderomotive force. Therefore, many physical mechanisms of the plasmas interact with ultra-intense laser pulse are different.

Modulational instability (MI) is one of the fundamental phenomena in the nonlinear interaction of an ultra-intense laser pulse with plasmas and has been discussed in various situations [5,6,10,11]. Recently, the MI of an intense right-hand elliptically polarized laser beam propagating through an e–p plasma is investigated [12]. In the present Letter, we investigate the MI of an ultra-intense linearly polarized laser pulse propagating in e–p plasmas by sidebands analysis [13]. First, taking into account of the properties of e–p plasmas, the wave equation is obtained. Then, the nonlinear dispersion relation and the growth rate of MI instability are derived. And last, the effects of plasmas number density, temperature, and laser intensity on the growth rate are analyzed by numerical method.

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2. Nonlinear wave equation

The wave equation for a laser pulse propagating in e–p plasmas is

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial}{\partial t}\mathbf{J}, \tag{1}$$

where $\mathbf{E}$ is the laser electric field, c is the speed of light in vacuum, and $\mathbf{J}$ is the plasma current density. Using $\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$ and $\mathbf{a} = \frac{e\mathbf{A}}{m_0 c^2}$, one can write the wave equation in terms of the normalized vector potential $\mathbf{a}$ as

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{a}}{\partial t^2} - \nabla^2 \mathbf{a} = \frac{4\pi}{c}\frac{e}{m_0 c^2}\mathbf{J}, \tag{2}$$

where m_0 is the rest mass of the electrons.

In the linearly polarized electromagnetic wave, the electron and positron velocities approximate to $\mathbf{v}_p = -c\mathbf{a}/\gamma_p$ and $\mathbf{v}_e = c\mathbf{a}/\gamma_e$, where $\gamma_{p,e} = \sqrt{1+|a|^2} \equiv \gamma$ is the Lorentz factor, and the subscript e (p) stands for the electrons (positrons). Therefore, the current density in the e–p plasmas can be written as

$$\mathbf{J} = -e(n_p + n_e)\frac{c}{\gamma}\mathbf{a}, \tag{3}$$

where n_p (n_e) is the positron (electron) number density. In the intense laser pulse, the low frequency of the perturbation of e–p plasmas density is determined by the balance equation [14]

$$k_B T_{p,e} \nabla \ln \frac{n_{p,e}}{n_0} = \nabla(\pm e\phi - \psi_P), \tag{4}$$

where k_B is the Boltzmann constant, T_e (T_p) is the temperature of electrons (positrons), n_0 is the unperturbed electron number density, and ψ_P is the ponderomotive potential. In the linearly polarized electromagnetic wave, the ponderomotive potential is given by $\psi_P = m_0 c^2 \gamma$. In Eq. (4), taking $+$ or $-$ represents the equation that is satisfied by electrons or positrons. The potential ϕ associated with the plasma slow motion is found from Poisson's equation

$$\nabla^2 \phi = 4\pi e(n_e - n_p). \tag{5}$$

Integrating Eq. (4) once, we obtain [5,6]

$$n_{p,e} = n_0 \exp\left[-\frac{m_0 c^2}{k_B T_{p,e}}(\gamma - 1) \pm \frac{e}{k_B T_{p,e}}\phi\right]. \tag{6}$$

Letting $n_{e,p} = n_0 N_{e,p}$ and $\phi = (m_0 c^2/e)\Phi$, Eq. (5) can be written in the dimensionless form $\frac{c^2}{\omega_p^2}\nabla^2\Phi = N_e - N_p$, where $\omega_p = \sqrt{4\pi n_0 e^2/m_0}$ is the plasmas frequency, $N_p = \exp[\beta_p(1-\gamma-\Phi)]$, $N_e = \exp[\beta_e(1-\gamma+\Phi)]$, $\beta_{e,p} = m_0 c^2/k_B T_{e,p}$, and the normalized ponderomotive potential takes the form $\Psi_P = \beta_{e,p}\gamma$. In the quasi-neutral limit $N_e = N_p$, we have [6]

$$\Phi = \frac{(1-\gamma)(\beta_p - \beta_e)}{\beta_p + \beta_e}. \tag{7}$$

Therefore,

$$N_e + N_p = 2N_e = 2\exp\left[\frac{2\beta_p\beta_e(1-\gamma)}{\beta_p+\beta_e}\right] = 2\exp\left[\beta(1-\gamma)\right], \tag{8}$$

where $\beta = \frac{2\beta_p\beta_e}{\beta_p+\beta_e}$ is the temperature parameter. Eq. (8) hints that in the ultra-intense laser pulse, the perturbation of the e–p plasma density is determined by the normalized ponderomotive potential.

Substituting Eqs. (3) and (8) into Eq. (2), one can obtains the wave equation of ultra-intense linearly polarized laser pulse propagating in e–p plasmas

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{a}}{\partial t^2} - \nabla^2 \mathbf{a} = -\frac{2\omega_p^2}{c^2}\frac{\exp[\beta(1-\gamma)]}{\gamma}\mathbf{a}, \tag{9}$$

where $\exp[\beta(1-\gamma)]$ includes the effects of ponderomotive force and thermal pressure, and γ relate to the relativistic effect.

3. Dispersion relation and modulational instability

The radiation field is assumed to consist of plane waves polarized in the x direction of the form $\mathbf{a} = (a_p + a_+ + a_-)\mathbf{e}_x$ [13], where a_p denotes the pump wave, $a_\pm$ are the anti-Stokes and Stokes sidebands, and $|a_\pm| \ll |a_p|$. The pump wave and the sidebands are given by

$$a_p = \frac{1}{2}a_0 \exp(ik_0 z - i\omega_0 t) + c.c., \tag{10}$$

$$a_+ = \frac{1}{2}\hat{a}_+ \exp\left[i(k_0+k)z - i(\omega_0+\omega)t\right] + c.c., \tag{11}$$

$$a_- = \frac{1}{2}\hat{a}_- \exp\left[i(k_0-k)z - i\left(\omega_0-\omega^*\right)t\right] + c.c., \tag{12}$$

where ω_0 and k_0 are the frequency and wave number of the pump, ω and k are the complex frequency and the wave number of the sidebands, and $*$ denotes the complex conjugate. The amplitude of the pump and of the sidebands are real and given by a_0 and $\hat{a}_\pm$, respectively.

Substituting Eqs. (10)–(12) into Eq. (9), and considering the lowest approximation, we obtain the nonlinear dispersion relation for a linearly polarized pump wave

$$\omega_0^2 - k_0^2 c^2 = 2\omega_p^2 P, \tag{13}$$

where $P = \frac{1}{\sqrt{1+|a_p|^2}}\exp[\beta(1-\sqrt{1+|a_p|^2})]$. In the low laser intensity limit ($a_0 \ll 1$), the relativistic effect is neglected, and Eq. (13) degenerated to

$$\omega_0^2 - k_0^2 c^2 = 2\omega_p^2. \tag{14}$$

Eq. (14) is consistent with the result in Ref. [15].

To analyze the MI, Eq. (9) is solved to the order $a_0^2 \hat{a}_\pm$, giving

$$D_+\hat{a}_+ = 2\omega_p^2 P\left[\hat{a}_+ - \frac{1+\beta\sqrt{1+|a_p|^2}}{(1+|a_p|^2)}\left(\frac{|a_p|^2}{2}\hat{a}_+ + \frac{|a_p|^2}{2}\hat{a}_-^*\right)\right], \tag{15}$$

$$D_-\hat{a}_-^* = 2\omega_p^2 P\left[\hat{a}_-^* - \frac{1+\beta\sqrt{1+|a_p|^2}}{(1+|a_p|^2)}\left(\frac{|a_p|^2}{2}\hat{a}_-^* + \frac{|a_p|^2}{2}\hat{a}_+\right)\right], \tag{16}$$

where $D_\pm = \omega_0^2 - k_0^2c^2 + \omega^2 - k^2c^2 \pm 2(\omega_0\omega - k_0kc^2)$. Combining Eqs. (15) and (16), we obtain the dispersion relation

$$\left[D_+ - 2\omega_p^2 P\left(1 - Q\frac{|a_p|^2}{2}\right)\right]\left[D_- - 2\omega_p^2 P\left(1 - Q\frac{|a_p|^2}{2}\right)\right] = 4\omega_p^4\left(PQ\frac{|a_p|^2}{2}\right)^2, \tag{17}$$

where $Q = \frac{1+\beta\sqrt{1+|a_p|^2}}{(1+|a_p|^2)}$. Using Eq. (13), the dispersion relation reduces to

$$\left(\omega^2 - k^2c^2\right)^2 - 4\left(\omega_0\omega - k_0kc^2\right)^2 + 2\omega_p^2|a_p|^2 PQ\left(\omega^2 - k^2c^2\right) = 0. \tag{18}$$

Considering $|\omega| \ll \omega_0$, $kc \ll \omega_0$, and $\omega_p^2 a_0^2 PQ \ll \omega_0$, we have