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# Geometric phase of a central qubit coupled to a spin chain in a thermal equilibrium state

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ABSTRACT

The geometric phase of a central qubit coupling to the surrounding XY chain in a transverse field at finite temperature is studied in this Letter. An explicit analytical expression of the geometric phase for coupled qubit is obtained in the weak coupling limit when the surrounding spin chain is in a thermal equilibrium state. It is shown that the GP displays dramatic change around the quantum phase transition points of the spin chain at zero and a finite range of temperature by numerical analysis. The result reveals that the GP can be used as a tool to detect QPT when the spin chain system is at finite temperature.



## 1. Introduction

A system can retain the information of its motion when it undergoes a cyclic evolution, in the form of geometric phases (GPs). It was first put forward by Pancharatnam in optics [1] and later studied explicitly by Berry in a general quantum system [2]. Since then, numerous generalizations have included nonadiabatic [3,4], noncyclic [4,5] and nonunitary [6,7] evolution, also for degenerate states [8]. Such a phase factor for the eigenstate depends only on the swept solid angle by the parameter vector in the parameter space, it may be less affected by uncontrolled fluctuations. Therefore, it can be utilized to implement geometric quantum computation (GQC) which is resilient to stochastic control errors [9–11]. However, because of the real situation, it is natural to extend the concept of GP from closed quantum systems to open quantum systems. Uhlman [6,7] first studied this issue as a purely mathematical problem. Then, based on the experimental context of quantum interferometry, Sjöqvist et al. [12] introduced a definition of GP for mixed states undergoing unitary evolution. In Ref. [13], Tong and coworkers developed a kinematic approach to the GP for open quantum systems in nonunitary evolution led by the coupling of environment.

Recently, the close relation between the geometric phase and quantum phase transitions (QPTs) in many body systems has been revealed [14–17]. Carollo and Pachos [14] showed that the geometric phase is much sensitive to control parameters of spin chain and can be used to detect the critical points of the spin chain. It is also shown that the connection of GPs with the typical features of QPTs such as the scaling feature, critical exponents and so on is not restricted to the XY spin chain model but universal to quantum many-body systems [15]. Yi and Wang [16] investigated the geometric phase induced in an auxiliary qubit by a one-dimensional XY spin chain and found that the GP changes sharply around the QPT point and tends to zero far from the critical points. Zhang et al. [17] studied the geometric phase of a qubit coupled to a XY spin chain with three-spin interaction and revealed the GP may be a tool to detect multi-spin interaction in a spin chain.

QPTs theoretically occur at absolute zero temperature ($T = 0$) due to abrupt changes in the qualitative properties of the ground state. At $T = 0$ there are no thermal fluctuations, QPTs are caused only by quantum fluctuations. By properly tuning the Hamiltonian parameter, such as an external magnetic field or a coupling constant, one can reach a special point, i.e., the critical point (CP), where the ground state of the system undergoes a dramatic change. QPTs strongly affect the macroscopic properties of the system. Some well-known examples of QPTs are the paramagnetic–ferromagnetic transition in some metals [18], the superconductor-transition [19], and superfluid–Mott insulator transition [20].

However, due to the third law of thermodynamics, it is difficult to achieve the absolute zero in practice, at this time thermal effects should be inevitable. Therefore, it is necessary to investigate the quantum system's property when both the quantum fluctuations and thermal fluctuations exist. In recent years, the influence

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of thermal effects on the quantum system's property in spin chains has been extensively investigated [21–23]. Zanardi et al. [22] have extended to finite temperature fidelity approach to quantum phase transitions and found mixed-state fidelity can indicate well QPTs at a finite temperature. Sadiek et al. [23] considered an infinite one-dimensional anisotropic XY spin chain with a nearest-neighbor time-dependent Heisenberg coupling $J(t)$ between the spins in presence of a time-dependent magnetic $h(t)$ at zero and finite temperatures, and found that the time evolution of entanglement in the system shows nonergodic and critical behavior.

In previous investigations, they have been shown that GPs of a qubit coupled to a spin chain change sharply around the critical points (CPs) of the spin chain and may be used to signal the critical points. Therefore, GPs are expected as a tool to detect QPTs of a many-body system [14–17]. In this Letter we are going to extend the geometric phase approach to finite temperature. We will consider a central qubit weakly coupled to a spin environment via a Heisenberg XY interaction. Our purpose is to investigate the geometric phase property of the qubit coupled to the spin chain, which is initially in the thermal equilibrium state, and expect the GP of the coupled qubit can indicate QPTs in a finite range of temperature. According to the definition of GP given by Ref. [13], we provide an explicit analytical expression of the geometric phase for the coupled qubit in the weak coupling limit. Our results show that the variation of the GP of the coupled qubit can well indicate the critical point phenomena in a finite range of temperature.

This Letter is organized as follows. In Section 2, the model under consideration is presented. In Section 3, an analytical expression of the GP is obtained in the weak coupling limit and the effect of the temperature on the GP is investigated. Finally, a short summary of the present investigations is given in Section 4.

## 2. Model

We consider a central qubit $\sigma_S^z$ transversely coupled to a spin environment via a Heisenberg XY interaction, and a transverse magnetic field is homogeneously applied to each spin of the chain. The qubit has a ground state $|g\rangle$ and a first excited state $|e\rangle$. We assume that there is no energy exchange between the qubit and the spin chain, the Hamiltonian of the whole system may read as follows (taking $\hbar = 1$ in the whole Letter):

$$H = H_S + H_E + H_{SE}, \tag{1}$$

where

$$H_S = \frac{\omega}{2}\sigma_S^z, \tag{2}$$

is the unperturbed Hamiltonian for the qubit,

$$H_E = -\sum_{l=1}^{L}\left(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z\right), \tag{3}$$

is the Hamiltonian of the transverse field XY spin chain, and

$$H_{SE} = -\sigma_S^z \otimes g\sum_{l=1}^{L}\sigma_l^z, \tag{4}$$

is the interaction between the qubit and the environment. Here $\omega$ is the transition frequency between the ground state and the excited state of the central qubit, $\gamma$ characterizes the anisotropy in the next-neighbor spin–spin interaction, $\lambda$ and $g$ are the transverse magnetic field strength and the coupling constant of the qubit to the spin chain, respectively. In above, $\sigma_S^z = |e\rangle\langle e| - |g\rangle\langle g|$ and $\sigma_l^\alpha$ ($\alpha = x, y, z$) are the Pauli matrices for spin at the $l$th site of the spin chain, and $L$ is the size of the spin chain. The anisotropy XY spin chain model contains the two special cases: for $0 < \gamma \leqslant 1$ it turns into a transverse Ising universality class XY spin chain, and it has a critical point at $|\lambda_c| = 1$; for $\gamma = 0$ it reduces to a transverse XX spin chain, which is critical for $|\lambda| \leqslant 1$. For the XY chain there exists a periodic boundary condition, i.e., $\sigma_1 = \sigma_{L+1}$. Here, we ignore the boundary terms [16].

After a transformation $(U_0^{-1}HU_0 - iU_0^{-1}\frac{dU_0}{dt})$, where $U_0 = \exp(-i\frac{\omega}{2}\sigma_S^z t)$ is the free evolution operator, we can remove the free-motion term of the central qubit in the Hamiltonian (1). Since $[H_S, H_{SE}] = 0$, an operator-valued parameter $\lambda_i = \lambda + g\sigma_S^z$ is a conversed quantity. Thus, $\lambda_i$ can be treated as a $c$ number. Obviously, $\lambda_i$ has two eigenvalues: $\lambda_i = \lambda + (-1)^i g$ $(i = 0, 1)$. In terms of the basis of the qubit, the Hamiltonian (1) can be rewritten as

$$H = \sum_{i=0,1}|i\rangle\langle i| \otimes H_{E,\lambda_i}. \tag{5}$$

Let us diagonalize the Hamiltonian $H_E + H_{SE}$. We follow the procedure presented in Ref. [24] by defining the conventional Jordan–Wigner (JW) transformation

$$\sigma_l^x = \prod_{n<l}\left(1 - 2c_n^+c_n\right)\left(c_l + c_l^+\right), \tag{6}$$

$$\sigma_l^y = -i\prod_{n<l}\left(1 - 2c_n^+c_n\right)\left(c_l - c_l^+\right), \tag{7}$$

$$\sigma_l^z = 1 - 2c_l^+c_l, \tag{8}$$

which maps spins to one-dimensional spinless fermions with creation (annihilation) operators $c_l^+$ ($c_l$), the Hamiltonian can be written into the form

$$H_{E,\lambda_i} = \sum_{l=1}^{L}\left[\left(c_l^+c_{l+1} + c_{l+1}^+c_l\right) + \gamma\left(c_l^+c_{l+1}^+ + c_{l+1}c_l\right) + \lambda_i\left(1 - 2c_l^+c_l\right)\right]. \tag{9}$$

Next we introduce Fourier transforms of the fermionic operators described by $d_k = \frac{1}{\sqrt{L}}\sum_l c_l e^{\frac{-i2\pi lk}{L}}$ with $k = -L/2, -L/2+1, \ldots, L/2-1$, the Hamiltonian Eq. (9) can be written in terms of the new fermionic operator $d_k$, $d_k^+$ as

$$H_{E,\lambda_i} = \sum_k\left[2(\cos k - \lambda_i)d_k^+d_k + i\gamma\sin k\left(d_k^+d_{-k}^+ - d_{-k}d_k\right)\right]. \tag{10}$$

And then using the Bogoliubov transformation by

$$b_{k,\lambda_i} = \cos\frac{\theta_{k,\lambda_i}}{2}d_k - i\sin\frac{\theta_{k,\lambda_i}}{2}d_{-k}^+, \tag{11}$$

with

$$\theta_{k,\lambda_i} = \arctan\frac{\gamma\sin\frac{2\pi k}{L}}{\lambda_i - \cos\frac{2\pi k}{L}}, \tag{12}$$

the Hamiltonian (10) can be diagonalized

$$H_{E,\lambda_i} = \sum_k \Omega_{k,\lambda_i}\left(b_{k,\lambda_i}^+b_{k,\lambda_i} - \frac{1}{2}\right), \tag{13}$$

where the energy spectrum

$$\Omega_{k,\lambda_i} = 2\sqrt{\left(\cos\frac{2\pi k}{L} - \lambda_i\right)^2 + \gamma^2\sin^2\frac{2\pi k}{L}}. \tag{14}$$

Since $g = 0$ and then $\lambda_i = \lambda$, thus, replacing $\lambda_i$ by $\lambda$ in (11)–(14), we can obtain the normal modes $a_{k,\lambda}$, $\theta_{k,\lambda}$, the diagonalized Hamiltonian and the corresponding eigenenergies of the pure spin chain. It is easily found that the relationship between the normal