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Remote state preparation using positive operator-valued measures

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ABSTRACT

We consider the process of remote state preparation using a pure state $|\psi\rangle$ with the maximal Schmidt number n. For any given state σ, pure or mixed, a construction of a positive operator-valued measure $\{M_j\}_{j=0}^n$ is provided. The classical outcome $j=0$ indicates the failure of a remote preparation of σ. All other classical outcomes $j>0$ correspond to unitary transformations of the receiver system such that σ can be prepared. The total probability of successful remote preparation depends on the state σ. Our protocol is a variation of conclusive teleportation and the classical bits required for this protocol are given by $\log_2(n+1)$, which is nearly half that of conclusive teleportation.

1. Introduction

Remote state preparation (RSP) [1,2] is a variation of quantum teleportation [3]. Both of these quantum communication protocols are designed to prepare a qubit state $|\phi\rangle$ in a remote quantum system without sending a quantum system carrying information about the quantum state $|\phi\rangle$, but instead sending classical information through classical channels. Both protocols require the maximally entangled qubit state previously shared by the sender (Alice) and receiver (Bob).

The main difference between these methods is that, for quantum teleportation, Alice knows nothing about $|\phi\rangle$, whereas for RSP Alice is allowed to know the state $|\phi\rangle$. The knowledge of $|\phi\rangle$ reduces the amount of resources required in RSP [1,2]. We know that the asymptotic classical communication cost of RSP is one bit per qubit, which is half that of quantum teleportation [4]. Unlike quantum teleportation, not all qubit states can be prepared successfully in the remote system.

Consider the case in which Alice and Bob share a non-maximally entangled pure qubit state. Mor [5] and Horodecki and Mor [6] have suggested the method of conclusive teleportation for quantum teleportation. In this approach, Alice makes a positive operator-valued measure (POVM) so that the unknown qubit state can still be teleported, but with a probability of less than one. The idea that both studies have proposed is applicable to finite-dimensional cases [8,9]. However, states can be prepared remotely and deterministically in Bob's system by using projection measurements [10].

POVMs can provide more information about quantum states than projective measurements can. Some tasks that cannot be performed using projective measurements can be completed using POVMs. For example, a set of non-orthogonal states cannot be distinguished using projective measurements, but can be discriminated unambiguously using POVMs [11,12]. This concept can be used to construct POVMs in conclusive teleportation [5,6]. Thus, determining what can be achieved by using POVMs in the RSP process yields compelling results. For example, Solís-Prosser and Neves proposed a strategy for RSP of spatial qubits by using a POVM that improves the probability of preparation and the fidelity and purity of the remote prepared states, as compared with the method of spatial postselection [7].

In this report, we assume that Alice and Bob share a pure state $|\psi\rangle$ with the maximal Schmidt number n. We provide a scheme to construct a POVM $\{M_j\}_{j=0}^n$ for a known state σ. The classical outcome $j=0$ denotes the failure of the remote preparation of $|\phi\rangle$, whereas the other classical outcomes $j>0$ indicate unitary transformation in Bob's system to prepare the state σ. Hence, the classical bits required to send the classical information are then given by $\log_2(n+1)$, which is nearly half of $\log_2(n^2+1)$ for conclusive teleportation [8,9]. The proposed protocol is a variation of conclusive teleportation such that Alice knows the state to prepare in Bob's system, and the communication cost is reduced because of the knowledge of the state σ.

As shown in [13], POVMs can be used to prepare any pure state remotely with a non-maximally entangled state shared previously. Our constructions of POVMs show that mixed states can also be prepared remotely, thus generalizing the results in [13]. The proposed approach is similar to the discussion of preparable ensembles for RSP by Kurucz and Adam [10]. They used the isomorphism between pure entangled states and antilinear operators [14,15],

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whereas the proposed method is based on a one-to-one correspondence between local operators with respect to the pure state $|\psi\rangle$ shared by Alice and Bob. This correspondence is a generalization of perfect correlation associated with maximally entangled states [16–18].

States with perfect correlation were first found by Einstein, Podolsky, and Rosen [19], and a two-dimensional example was provided by [20]. In a state with perfect correlation, with an outcome of an observable of one system, the outcome of an observable of the other system can be predicted with certainty. Thus, it establishes a one-to-one correspondence between local observables. This one-to-one correspondence can be generalized to that between local operators with respect to states with maximal Schmidt number. By generalizing this correspondence, the RSP process becomes clear. Conditions for using unitary transformations to prepare σ corresponding to different classical outcomes j are found. The POVM for the remote preparation of σ is then easy to construct. The optimal total probability of the successful preparation of σ is also found. The proposed method simplifies the method of the joint RSP [21], in which the information about the pure state to be prepared remotely is considered in two parts: the real coefficients and phases. Moreover, our strategy can be used to prepare any mixed state remotely.

2. One-to-one correspondence between local operators

According to [16,17], a state ρ on $\mathcal{B}(\mathcal{H}^{\mathcal{A}})\otimes\mathcal{B}(\mathcal{H}^{\mathcal{B}})$ is said to have perfect correlation if, for $A=A^\dagger\in\mathcal{B}(\mathcal{H}^{\mathcal{A}})$, there is $B=B^\dagger\in\mathcal{B}(\mathcal{H}^{\mathcal{B}})$ such that

$$\mathrm{Tr}\big(\rho(A\otimes\mathbb{I}-\mathbb{I}\otimes B)^2\big)=0, \tag{1}$$

and vice versa. Eq. (1) means that the joint distribution of A and B with respect to ρ is concentrated on the diagonal. For finite-dimensional systems, a state with perfect correlation is a maximally entangled state Ω [18]. Consequently, Eq. (1) implies that $A\otimes\mathbb{I}\Omega=\mathbb{I}\otimes B\Omega$, leading to a one-to-one mapping $A\mapsto B$ with respect to Ω. We know that, for a state $|\psi\rangle$ with the maximal Schmidt number n, there is a maximally entangled state $|\Psi\rangle$ with the same Schmidt basis. As follows, the one-to-one mapping between local observables with respect to the maximally entangled state $|\Psi\rangle$ can be found explicitly and generalized to that between local operators with respect to the state $|\psi\rangle$.

Let $\mathcal{H}^{\mathcal{A}}$ and $\mathcal{H}^{\mathcal{B}}$ be n-dimensional Hilbert spaces held by Alice and Bob. Suppose that Alice and Bob share a pure state $|\psi\rangle$ on $\mathcal{H}^{\mathcal{A}}\otimes\mathcal{H}^{\mathcal{B}}$ with the Schmidt representation of the form

$$|\psi\rangle=\sum_{j=1}^{n}\rho_j^{1/2}|e_j\rangle|f_j\rangle \tag{2}$$

where $\sum_{j=1}^n\rho_j=1$, $\rho_j\neq 0$ for $j=1,\dots,n$ and $\{|e_j\rangle\}_{j=1}^n$, $\{|f_j\rangle\}_{j=1}^n$ are orthonormal bases for $\mathcal{H}^{\mathcal{A}}$, $\mathcal{H}^{\mathcal{B}}$. Define an antiunitary operator J on $\mathcal{H}^{\mathcal{A}}\otimes\mathcal{H}^{\mathcal{B}}$ as follows:

$$J\sum_{j,k=1}^{n}\lambda_{jk}|e_jf_k\rangle=\sum_{j,k=1}^{n}\overline{\lambda_{jk}}|e_kf_j\rangle. \tag{3}$$

J is called the modular conjugation associated with $|\psi\rangle$ [22,23], and has the interesting properties: $J=J^{-1}=J^\dagger$. Because $\{|e_j\rangle\}_1^n$ and $\{|f_j\rangle\}_1^n$ are orthonormal bases for $\mathcal{H}^{\mathcal{A}}$ and $\mathcal{H}^{\mathcal{B}}$, every $A\in\mathcal{B}(\mathcal{H}^{\mathcal{A}})$ and $B\in\mathcal{B}(\mathcal{H}^{\mathcal{B}})$ have the forms: $A=\sum_{j,k=1}^n\lambda_{jk}|e_j\rangle\langle e_k|$ and $B=\sum_{j,k=1}^n\mu_{jk}|f_j\rangle\langle f_k|$. The mapping $X\mapsto JXJ$ with $X\in\mathcal{B}(\mathcal{H}^{\mathcal{A}}\otimes\mathcal{H}^{\mathcal{B}})$ establishes a one-to-one correspondence between two local algebras $\mathcal{B}(\mathcal{H}^{\mathcal{A}})$ and $\mathcal{B}(\mathcal{H}^{\mathcal{B}})$ as follows:

$$J\left(\sum_{j,k=1}^{n}\lambda_{jk}|e_j\rangle\langle e_k|\otimes\mathbb{I}\right)J=\mathbb{I}\otimes\sum_{j,k=1}^{n}\overline{\lambda_{jk}}|f_j\rangle\langle f_k|. \tag{4}$$

Define the following mappings $j_{\mathcal{A}}:\mathcal{B}(\mathcal{H}^{\mathcal{A}})\to\mathcal{B}(\mathcal{H}^{\mathcal{B}})$ and $j_{\mathcal{B}}:\mathcal{B}(\mathcal{H}^{\mathcal{B}})\to\mathcal{B}(\mathcal{H}^{\mathcal{A}})$ by

$$j_{\mathcal{A}}(A)=\sum_{j,k=1}^{n}\overline{\lambda_{jk}}|f_j\rangle\langle f_k|, \tag{5}$$

$$j_{\mathcal{B}}(B)=\sum_{j,k=1}^{n}\overline{\mu_{jk}}|e_j\rangle\langle e_k| \tag{6}$$

for $A=\sum_{j,k=1}^n\lambda_{jk}|e_j\rangle\langle e_k|$ and $B=\sum_{j,k=1}^n\mu_{jk}|f_j\rangle\langle f_k|$. Thus, Eq. (4) becomes

$$J(A\otimes\mathbb{I})J=\mathbb{I}\otimes j_{\mathcal{A}}(A), \tag{7}$$

and similarly for $j_{\mathcal{B}}$.

Clearly, $j_{\mathcal{A}}$ is an antilinear *-isomorphism and has the following properties:

$$j_{\mathcal{A}}(\lambda A)=\bar{\lambda}j_{\mathcal{A}}(A), \tag{8}$$

$$j_{\mathcal{A}}(A^\dagger)=j_{\mathcal{A}}(A)^\dagger, \tag{9}$$

$$j_{\mathcal{A}}(A_1A_2)=j_{\mathcal{A}}(A_1)j_{\mathcal{A}}(A_2) \tag{10}$$

for $A,A_1,A_2\in\mathcal{B}(\mathcal{H}^{\mathcal{A}})$. The inverse of $j_{\mathcal{A}}$ is given by $j_{\mathcal{A}}^{-1}=j_{\mathcal{B}}$.

Let $|\Psi\rangle=\sum_{j=1}^n|e_j\rangle|f_j\rangle$ be the non-normalized maximally entangled state associated with $|\psi\rangle$. The perfect correlation of $|\Psi\rangle$ can be directly expressed by $j_{\mathcal{A}}$ and $j_{\mathcal{B}}$ in the following way: from (4) it holds that

$$A\otimes\mathbb{I}|\Psi\rangle=\mathbb{I}\otimes j_{\mathcal{A}}(A^\dagger)|\Psi\rangle,\qquad \mathbb{I}\otimes B|\Psi\rangle=j_{\mathcal{B}}(B^\dagger)\otimes\mathbb{I}|\Psi\rangle \tag{11}$$

for local operators $A\in\mathcal{B}(\mathcal{H}^{\mathcal{A}})$, $B\in\mathcal{B}(\mathcal{H}^{\mathcal{B}})$. More precisely, the operator $j_{\mathcal{A}}(A^\dagger)$ (or $j_{\mathcal{B}}(B^\dagger)$) is the only operator B (or A) in $\mathcal{B}(\mathcal{H}^{\mathcal{B}})$ (or $\mathcal{B}(\mathcal{H}^{\mathcal{A}})$) that satisfies $A\otimes\mathbb{I}|\Psi\rangle=\mathbb{I}\otimes B|\Psi\rangle$, respectively. For self-adjoint operators $A=A^\dagger\in\mathcal{B}(\mathcal{H}^{\mathcal{A}})$ and $B=B^\dagger\in\mathcal{B}(\mathcal{H}^{\mathcal{B}})$, (11) implies

$$\langle\Psi|\big(A\otimes\mathbb{I}-\mathbb{I}\otimes j_{\mathcal{A}}(A)\big)^2|\Psi\rangle=0,$$
$$\langle\Psi|\big(\mathbb{I}\otimes B-j_{\mathcal{B}}(B)\otimes\mathbb{I}\big)^2|\Psi\rangle=0. \tag{12}$$

This in turn implies that the joint distribution of A and $j_{\mathcal{A}}(A)$ with respect to $|\Psi\rangle\langle\Psi|$ is concentrated on the diagonal, and similarly for B and $j_{\mathcal{B}}(B)$. Hence, according to the outcome of A (or B), the outcome of $j_{\mathcal{A}}(A)$ (or $j_{\mathcal{B}}(B)$) can be predicted with certainty [16,17]. Eq. (12) demonstrates the perfect correlation of $|\Psi\rangle$ [17,18]. Thus, the mappings $A\mapsto j_{\mathcal{A}}(A^\dagger)$ and $B\mapsto j_{\mathcal{B}}(B^\dagger)$ represent the property of the perfect correlation of $|\Psi\rangle$.

Eq. (11) can be generalized in the following manner for a non-maximally entangled pure state $|\psi\rangle$ (2) with the maximal Schmidt number n. Let $\rho^{\mathcal{A}}=\sum_{j=1}^n\rho_j|e_j\rangle\langle e_j|$, $\rho^{\mathcal{B}}=\sum_{j=1}^n\rho_j|f_j\rangle\langle f_j|$ be the reduced states of $|\psi\rangle\langle\psi|$ on $\mathcal{H}^{\mathcal{A}}$, $\mathcal{H}^{\mathcal{B}}$, respectively. Because $|\psi\rangle$ has the Schmidt number n, both $\rho^{\mathcal{A}}$ and $\rho^{\mathcal{B}}$ are invertible. The relationships between $|\psi\rangle$ and $|\Psi\rangle$ can be expressed as

$$|\Psi\rangle=\big(\rho^{\mathcal{A}}\big)^{-1/2}\otimes\mathbb{I}|\psi\rangle=\mathbb{I}\otimes\big(\rho^{\mathcal{B}}\big)^{-1/2}|\psi\rangle. \tag{13}$$

Substituting (13) into (11) leads to

$$A\otimes\mathbb{I}|\psi\rangle=\mathbb{I}\otimes T_{\mathcal{A}}(A)|\psi\rangle,\qquad B\otimes\mathbb{I}|\psi\rangle=T_{\mathcal{B}}(B)\otimes\mathbb{I}|\psi\rangle \tag{14}$$

with