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Delayed feedback control of chaos in a switched arrival system

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Abstract

In this Letter, the problem of controlling chaos in switched arrival systems is considered. Poincaré section and Poincaré mapping are introduced for defining periodic orbits of this hybrid system. A delayed impulsive feedback method is proposed for stabilizing unstable long periodic orbits embedded in the chaotic attractor. The method does not rely on a priori knowledge of the position of the orbit to be stabilized. The convergence of the proposed control algorithm is proved.

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Keywords: Chaotic hybrid systems; Switched arrival system; Unstable periodic orbit; Controlling chaos; Delayed feedback

1. Introduction

As a prototype of hybrid systems, switched flow models are very suitable for describing scheduling of many manufacturing systems and computer systems where a large amount of work is processed at a unit time [1,6]. Switched flow systems are classified into two types, namely switched arrival systems and switched server systems [1]. The dynamical behavior of such hybrid systems can be very complex. It was shown in [1] and [3] that switched arrival systems can be chaotic while switched server systems are eventually periodic.

As it is well known, a chaotic attractor includes infinitely many unstable periodic orbits in general. Stabilizing unstable periodic orbits embedded in chaos has drawn much attention and has become a very active multidisciplinary research area. The first chaos control method, known as OGY method proposed by Ott et al. [5], stabilizes unstable periodic orbits using a small discontinuous parameter perturbation. Although the method is effective, it requires information about inherent unstable periodic orbits which are used as reference signals for control. Because of the unstable nature of the inherent orbits in chaos, these signals are very difficult to obtain in general if not possible. Pyra-

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gas proposed a delayed feedback control method for stabilizing inherent unstable periodic orbits in chaotic systems [7]. The advantage of this method is it requires only a period constant instead of the exact information of the orbit to be stabilized. Further extension of the method has been reported [8,11]. However, due to the hybrid property of the switched arrival system, it may be difficult to apply these methods while both the system and the target orbit are nondifferentiable at transition points.

To stabilize an unstable periodic orbit embedded in the chaotic attractor of the switched arrival system, Ushio et al. proposed a method which changes the limited continuous processing time [12]. Recently, Rem et al. discussed the possibility of controlling the system to a periodic orbit by introducing a reduced-production regime into the system scheduling [10]. These methods, however, require a given orbit as the reference and therefore rely upon the knowledge of position of the orbit to be stabilized, which is generally difficult to obtain for long periodic orbits of chaotic systems. Only period-one orbit of the switched arrival system has been considered so far.

In this Letter, the concept of Poincaré section and Poincaré mapping are introduced into the switched arrival system. A systematic method is given for describing periodic orbits of this hybrid system. Then we propose a delayed impulsive feedback method for designing the control strategy with the purpose of stabilizing the system at an unstable (long) periodic orbit embedded in the chaotic attractor of the system. The convergence analysis of the proposed algorithm is also provided.

2. Switched arrival system

Consider a system of N buffers and one server. Each buffer corresponds to a machine processing work. Work is removed from buffer i at a fixed rate of $\rho_i > 0$ while the server delivers material to a selected buffer at a unit rate. The system is assumed to be closed so that $\sum_{i=1}^{N} \rho_i = 1$. Fig. 1 shows a switched arrival system for $N = 3$ with the server on buffer 1.

Let $x_i(t)$ be the amount of work in buffer i at the time $t \geqslant 0$. We assume that the initial value of the total amount of work is 1, that is $\sum_{i=1}^{N} x_i(0) = 1$, which implies that $\sum_{i=1}^{N} x_i(t) = 1$ for any $t \geqslant 0$. We use

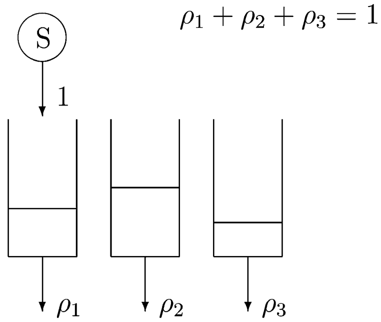

Fig. 1. Switched arrival system.

$(q(t), x(t))$ to denote the hybrid state of the switching arrival system, where the discrete state $q(t) \in Q = \{1, 2, \ldots, N\}$ denotes the buffer to which the server is delivering material at $t \geqslant 0$, and $x(t) = [x_1(t), \ldots, x_N(t)]^T \in X \subseteq R^N$ is the continuous state of the system. In future the equation $(q(t), x(t)) = (q'(t), x'(t))$ will imply that $q(t) = q'(t)$ and $x(t) = x'(t)$. A transition map $R(q(t), x(t)) = (q'(t), x'(t))$ is used in this Letter to denote that the system is switched from (q, x) to (q', x') at the time t. Note that in the switched arrival system a switching action does not change the work distribution in buffers, i.e., $R(q, x) = (q', x)$. A guard of the switching system is defined as

$$G(q, q') = \big\{x \in X \subseteq R^N\colon\ R(q, x) = (q', x),\\ q \in Q, q' \in Q, q \neq q'\big\}, \tag{1}$$

which is actually the set of transition states at which the server will switch from buffer q to buffer q'.

The location of the server is a control variable in the discussed system, and may be selected using a feedback policy. An *elementary control policy* considered in [1] is to define the guard as

$$G_e(i, j) = \big\{x \in X \subseteq R^N\colon\ x_j = 0\big\}, \tag{2}$$

which implies that the server switches buffers when some buffer becomes empty. Since $G_e(i, j)$ is irrelevant to i, we will write it as $G_e(*, j)$. Under this control policy we have $x_i(t) \geqslant 0$ for all $i \in Q$ and all $t \geqslant 0$. So the system domain

$$X = \left\{x\colon\ \sum_{i=1}^{N} x_i = 1, x_i \geqslant 0\right\} \tag{3}$$

is a polytope in R^N. For $N = 3$ it is an equilateral triangle as shown in Fig. 2. And in this case, obviously,