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Controlling fractional order chaotic systems based on Takagi–Sugeno fuzzy model and adaptive adjustment mechanism

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ABSTRACT

In this Letter, a kind of novel model, called the generalized Takagi–Sugeno (T-S) fuzzy model, is first developed by extending the conventional T-S fuzzy model. Then, a simple but efficient method to control fractional order chaotic systems is proposed using the generalized T-S fuzzy model and adaptive adjustment mechanism (AAM). Sufficient conditions are derived to guarantee chaos control from the stability criterion of linear fractional order systems. The proposed approach offers a systematic design procedure for stabilizing a large class of fractional order chaotic systems from the literature about chaos research. The effectiveness of the approach is tested on fractional order Rössler system and fractional order Lorenz system.

1. Introduction

Although a fractional derivative is a mathematical topic with more than 300 years old history, its application to physics and engineering is just a recent focus of interest. Many systems are known to display fractional order dynamics, such as viscoelastic systems [1], dielectric polarization [2], electrode–electrolyte polarization [3], and electromagnetic waves [4]. Meanwhile, it has been shown that many fractional order differential systems can demonstrate chaotic behavior, such as the fractional order Lorenz system [5], fractional order Chua circuit [6], fractional order Rössler system [7], fractional order Chen system [8], fractional order unified system [9], and so on.

Chaos control, as a new direction of chaos research, has gained increasing attention in the past two decades in order to avoid troubles arising from unusual behaviors of a chaotic system. An important objective of chaos control is to suppress the chaotic oscillations completely or reduce them to the regular oscillations. Since the pioneering work of Ott, Grebogi and Yorke [10], many control techniques have been implemented in the control of chaotic systems, for example, the OGY method [10], impulsive control methods [11–13], linear state space feedback methods [14,15] and adaptive adjustment mechanism [16,17], among many others. However, to the best of the authors' knowledge, the research on the control of chaotic systems has mainly focused on integer order chaotic systems, and the corresponding research on fractional order chaotic systems has received very little attention despite its practical significance.

On the other hand, fuzzy control has also been applied to stabilizing chaotic systems since the T-S fuzzy model can express a chaotic system with a small number of implications of rules. An approach to control chaos via the linear matrix inequality (LMI) based fuzzy control system design has been suggested in Refs. [18,19], where the key idea is to use the well-known T-S fuzzy model to represent typical chaotic systems and then design a controller for the fuzzy model.

In the present Letter, a kind of novel model, called the generalized T-S fuzzy model, is first developed by extending the conventional T-S fuzzy model. Then, a new method is developed to stabilize a large class of fractional order chaotic systems via the generalized T-S fuzzy model and AAM proposed by Huang [16,17], which was extended by Bu et al. [20] and Zheng [21]. The approach represents a fractional order chaotic system by fractional order linear systems in different state space regions based on the generalized T-S fuzzy model and then stabilizes the fractional order linear systems in different state space regions by adaptive adjustment mechanism. Sufficient conditions are derived to guarantee chaos control from the stability criterion of linear fractional order systems. The rest of this Letter is organized as follows. In Section 2, the Caputo definition of fractional derivative and generalized T-S fuzzy model are given. In Section 3, based on generalized T-S fuzzy model and AAM, we discuss the control of fractional order chaotic

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systems. Some simple but generic criteria of the control of fractional order chaotic systems are established. Simulation study is given in Section 4, and conclusions are drawn in Section 5.

2. Caputo definition of fractional derivative and generalized T-S fuzzy model

2.1. Caputo definition of fractional derivative

In the following, the Caputo definition of fractional derivative [22] is introduced.

$$D_*^{\alpha}x(t) = J^{m-\alpha}x^{(m)}(t) \quad \text{with } \alpha > 0, \tag{1}$$

where $m = [\alpha]$, i.e., m is the first integer which is not less than α, $x^{(m)}$ is the m-order derivative in the usual sense, and J^{β} ($\beta > 0$) is the β-order Reimann–Liouville integral operator with expression:

$$J^{\beta}y(t) = \frac{1}{\Gamma(\beta)}\int_0^t (t-\tau)^{\beta-1}y(\tau)\,d\tau, \tag{2}$$

where Γ stands for Gamma function, and the operator D_*^{α} is generally called the α-order Caputo differential operator.

A fractional order linear time-invariant system can be represented by the following state space model

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}} = Ax(t) \quad \text{with } x(0) = x_0, \tag{3}$$

where $x(t) \in R^n$, $A \in R^{n\times n}$, α is the fractional order. It has been shown that autonomous system (3) is asymptotically stable if and only if the following condition is satisfied

$$\left|arg\big(eig(A)\big)\right| > \frac{\alpha\pi}{2}, \tag{4}$$

where $0 < \alpha < 1$ and $eig(A)$ represents the eigenvalues of matrix A. Also, this system is stable if and only if $|arg(eig(A))| \geqslant \frac{\alpha\pi}{2}$ and those critical eigenvalues that satisfy $|arg(eig(A))| = \frac{\alpha\pi}{2}$ have geometric multiplicity of one.

2.2. Generalized T-S fuzzy model

The generalized T-S fuzzy model is obtained by extending the conventional T-S fuzzy model [23]. In the generalized T-S fuzzy model, local dynamics in different state space regions are represented by fractional order linear models. The overall model of the system is achieved by fuzzy blending of these fractional order linear models. Suppose that the generalized T-S fuzzy model is given in the following form:

Rule i: IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_p(t)$ is M_{ip}

$$\text{THEN } \frac{d^{\alpha}x(t)}{dt^{\alpha}} = A_i x(t),\ i = 1, 2, \ldots, r, \tag{5}$$

where M_{ij} ($j = 1, 2, \ldots, p$) is the fuzzy set and r is the number of IF–THEN rules, $x(t) \in \Re^n$ is the state vector, $A_i \in \Re^{n\times n}$, $z_1(t) \sim z_p(t)$ are the premise variables and α ($0 < \alpha < 1$) is the fractional order. The final output of the generalized T-S fuzzy model is inferred as follows:

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}} = \frac{\sum_{i=1}^r w_i(z(t))A_i x(t)}{\sum_{i=1}^r w_i(z(t))}, \tag{6}$$

where $z(t) = (z_1(t), z_2(t), \ldots, z_p(t))$,

$$w_i\big(z(t)\big) = \prod_{j=1}^p M_{ij}\big(z_j(t)\big) \tag{7}$$

for all t, with $M_{ij}(z(t))$ being the grade of membership of $z_j(t)$ in M_{ij}, satisfying the following conditions:

$$\begin{cases} \sum_{i=1}^r w_i\big(z(t)\big) > 0, \\ w_i\big(z(t)\big) \geqslant 0, \quad i = 1, 2, \ldots, r. \end{cases}$$

By introducing $h_i(z(t)) = w_i(z(t))/\sum_{i=1}^r w_i(z(t))$ instead of $w_i(z(t))$, the expression (6) is rewritten as

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}} = \sum_{i=1}^r h_i\big(z(t)\big)A_i x(t). \tag{8}$$

Note that

$$\begin{cases} \sum_{i=1}^r h_i\big(z(t)\big) = 1, \\ h_i\big(z(t)\big) \geqslant 0, \quad i = 1, 2, \ldots, r, \end{cases} \tag{9}$$

for all t, where $h_i(z(t))$ can be regarded as the normalized weights of the IF–THEN rules.

3. Controlling fractional order chaotic systems based on generalized T-S fuzzy model and AAM

Suppose that a fractional order chaotic system can be exactly represented by the generalized T-S fuzzy model (5). In order to stabilize the generalized T-S fuzzy model (5), by AAM we denote the following adjusted system:

Rule i: IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_p(t)$ is M_{ip}

$$\text{THEN } \frac{d^{\alpha}x(t)}{dt^{\alpha}} = (I-\Gamma)A_i x(t) - \Gamma x(t),\ i = 1, 2, \ldots, r. \tag{10}$$

The final output of the adjusted system (10) is inferred as follows:

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}} = \sum_{i=1}^r h_i\big(z(t)\big)\big[(I-\Gamma)A_i x(t) - \Gamma x(t)\big], \tag{11}$$

where $\Gamma = diag\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ is an adaptive parameter matrix and $\gamma_j \in [0, 1]$, $j = 1, 2, \ldots, n$ are referred to as the adaptive parameter named after Refs. [16,17]. It is easy to see that the adjusted system (10) has the same equilibrium point $x = 0$ as that of the system (5).

The following theorem gives a sufficient condition for the stability of the adjusted system (10) and a convenient guide to the choice of the adaptive parameters.

Theorem 1. *Let $G = (I-\Gamma)A_1 - \Gamma$, $\Gamma = diag\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ and $\gamma_j \in [0, 1]$, $j = 1, 2, \ldots, n$. If there exists an adaptive parameter matrix Γ, such that $(I-\Gamma)A_1 = (I-\Gamma)A_i$, $i = 2, 3, \ldots, r$ and $|arg(eig(G))| > \frac{\alpha\pi}{2}$, then the adjusted system (10) is globally asymptotically stable about the equilibrium point $x = 0$, that is, the chaotic orbits of the original fractional order chaotic system (5) can be stabilized to the equilibrium point $x = 0$ by AAM.*

Proof. If the conditions $(I-\Gamma)A_1 = (I-\Gamma)A_i$, $i = 2, 3, \ldots, r$ hold, the fractional order chaotic systems represented by (10) or (11) are exactly linearized as

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}} = Gx(t). \tag{12}$$

Therefore, from stability theory of fractional order linear time-invariant system, we can conclude that system (12) is globally