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Quantum entanglement, indistinguishability, and the absent-minded driver's problem

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Abstract

The absent-minded driver's problem illustrates that probabilistic strategies can give higher pay-offs than deterministic ones. We show that there are strategies using quantum entangled states that give even higher pay-offs, both for the original problem and for the generalized version with an arbitrary number of intersections and any possible set of pay-offs.
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1. Introduction

1.1. The absent-minded driver's problem

The so-called *paradox of the absent-minded driver* was introduced by Piccione and Rubinstein in [1] and further discussed in [2,3] and references therein: an individual is sitting late at night in a bar planning his midnight trip home. The trip starts at the bar, the START in Fig. 1. There is a highway with two consecutive exits (or intersections), X and Y, and he has to take the second, Y, to get home (pay-off 4). If he takes the first one, he arrives at a bad neighborhood (pay-off 0), and if he fails to take either, he has to stay in a motel at the end of the highway (pay-off 1). He cannot go back. There are two essential assumptions:

(I) *Indistinguishability*: The intersections X and Y are indistinguishable by any experiment performed at one intersection. When the driver is at one intersection, no experiment can give him information about which intersection he is at.

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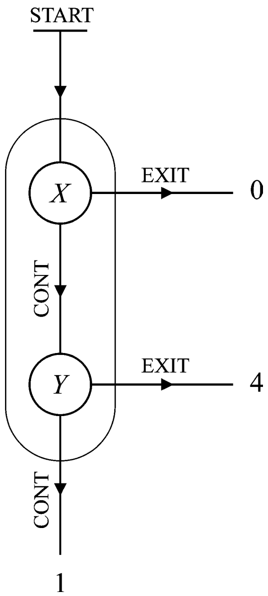

Fig. 1. The absent-minded driver's problem.

(II) *Absent-mindedness*: The driver is absent-minded and is aware of this fact. Absent-mindedness only affects his memories about whether he has already gone through one of the intersections; at X he knows he might be at Y but has forgotten passing X, and at Y he cannot remember passing X. Apart from this, the driver is perfectly able to remember anything else.

Some remarks about these assumptions follow:

(i) These assumptions are not independent: the indistinguishability of the intersections is only relevant for an absent-minded driver, and absent-mindedness is only relevant when the intersections are indistinguishable (otherwise, the driver could obtain information for decision-making in spite of his absent-mindedness).

(ii) Since the driver's absent-mindedness is limited to his memories about the intersections, but does not prevent him from possessing information about the rest of the universe, then it is forbidden any experiment at one intersection whose result, together with any information about the rest of the universe, allows the driver to obtain information about which intersection he is at.

(iii) Implicit in the rules is the fact that the driver cannot transmit information from one intersection to the rest of the universe (and, in particular, to the other intersection), because this could be used to distinguish the intersections.

The above scenario allows Piccione and Rubinstein to exhibit a conflict between two ways of reasoning at an intersection:

> "Planning his trip at the bar, the decision maker must conclude that it is impossible for him to get home and that he should not exit when reaching an intersection. Thus, his optimal plan will lead him to spend the night at the motel and yield a payoff of 1. Now suppose that he reaches an intersection. If he had decided to exit, he would have concluded that he is at the first intersection. Having chosen the strategy to continue, he concludes that he is at the first intersection with probability $1/2$. Then, reviewing his plan, he finds that it is optimal for him to leave the highway since it yields an expected payoff of 2. Despite no new information and no change in his preferences, the decision maker would like to change his initial plan once he reaches an intersection! [1]"

Piccione and Rubinstein make use of this apparent paradox to illustrate the advantages of probabilistic (or *random* [1], or *mixed*) strategies versus deterministic (or *pure* [1]) strategies. At the intersections, the driver can either CONTINUE along or EXIT the highway. Accordingly, there are two possible deterministic strategies: either to always CONTINUE (pay-off 1) or to always EXIT (pay-off 0). Alternatively, at the intersections, the driver can toss a (suitable weighted) coin with a probability p for heads (which means CONTINUE) and a probability $1 - p$ for tails (which means EXIT). The expected pay-off of this probabilistic strategy is $4p(1-p)+p^2$. Therefore, if $p > 1/3$, this strategy gives a higher pay-off than the best deterministic strategy. The optimal probabilistic strategy consists of choosing $p = 2/3$ (pay-off $4/3$).

1.2. Quantum strategies

Game theory has recently found a new direction based on the possibility of the resources of quantum

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