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Parameters identification and adaptive synchronization of chaotic systems with unknown parameters

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Abstract

Based on Lyapunov stabilization theory, an adaptive controller with parameters identification for a class of chaotic systems with unknown parameters is proposed in this Letter. The proposed control scheme is successfully applied to some typical chaotic systems, which can be spilt into two terms: one is the term with known states, the other is the symmetric matrix term with unknown parameters, such as Lorenz system. And with the proposed adaptive control law, the two unified systems with unknown parameter are also to be synchronized. Simulation results verify the proposed scheme's effectiveness.

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Keywords: Adaptive control; Parameters identification; Lyapunov stabilization theory; Chaotic synchronization; Unified system

1. Introduction

A chaotic system is a nonlinear deterministic system and its prominent characteristic is the sensitive dependence on initial conditions, that is, the trajectory of the chaotic system from different initial condition will be varying greatly after the transition time. In these years, the investigations in chaos control [1] and chaos synchronization [2] have been developed extensively since Pecora and Carroll's original research work [3]. Especially chaos synchronization has been investigated very intensely because of all its potential application in communication and information processing [4]. Since the chaotic system is a special nonlinear system, many nonlinear control methods can be used to control and synchronize chaotic system, such as adaptive control [5], variable structure control [6], adaptive backstepping control [7–9], states feedback control [10], active control [11–14] and so on. But the most of aforesaid methods are invalid when the parameters of chaotic systems are unknown. Recently, a lot of effort has been devoted to it. The adaptive synchronization

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technique by Wang et al. [15] works for Chen system with unknown parameters. Li et al. [16] derived the adaptive controller based on Lyapunov stability theory by introducing the update law to synchronize uncertain Rössler and Chen systems. Zhang and Ma [17,18] synchronize uncertain chaotic systems with parameters perturbation via active control and active sliding mode control.

In this Letter an adaptive controller with parameters identification based on Lyapunov stabilization theory for a class of uncertain chaotic systems, which can be decomposed into two terms: one is the term with known states, the other is the symmetric matrix term with unknown parameters. Many famous chaotic systems have this special structure such as Lorenz system, Lü system, which will be discussed in detail in the simulation examples. Also with the proposed adaptive control law, the two unified systems with unknown parameter are also to be synchronized and the unknown parameter converge to its true value finally. The simulation results show that the proposed technique is effective.

2. Design of adaptive controller with parameters identification

Considering an uncertain chaotic system with state equation in the form

$$\dot{x} = f(x) + F(x)\theta, \tag{1}$$

where $x \in R^n$ is the state vector, $f \in R^n$, $F \in R^{n\times n}$, and F is a symmetric matrix, $\theta \in R^n$ is an unknown parameter vector.

Remark. The class of nonlinear dynamical systems includes an extensive variety of chaotic systems such as Lorenz system and Lü system.

Eq. (1) is considered as a drive system. A response system is given by

$$\dot{y} = f(y) + F(y)\hat{\theta} + u, \tag{2}$$

where $\hat{\theta} \in R^n$ represents the estimate of unknown parameter vector θ, and $u \in R^n$ is a controller.

Subtracting Eq. (1) from (2), the following error system equation is formed:

$$\dot{e} = \tilde{f} + F(x)\tilde{\theta} + \tilde{F}\hat{\theta} + u, \tag{3}$$

where $e = y - x$, $\tilde{f} = f(y) - f(x)$, $\tilde{F} = F(y) - F(x)$, and $\tilde{\theta} = \hat{\theta} - \theta$ is a parameter error vector.

Let

$$u = -\tilde{f} - \tilde{F}\hat{\theta} + Ae, \tag{4}$$

where the matrix A is chosen such that it has all its eigenvalues on the left-hand side of the complex plane. Then Eq. (3) can be formulated as

$$\dot{e} = Ae + F(x)\tilde{\theta}. \tag{5}$$

In the theory of control, Lyapunov functions can be used to demonstrate the stability of some state points of a system. With reference to the study of temporal evolution of the synchronization error e, the Lyapunov function $V(e)$ can be defined as a continuously differentiable real valued function with the following properties:

(1) $V(e) > 0$ for all $e \neq 0$ and $V(e) = 0$ for $e = 0$.
(2) $\dot{V}(e) < 0$ for all $e \neq 0$.

If for system (5) one can find a Lyapunov function, then the error e is globally stable.

Theorem. *If there exists a positive symmetric matrix P such that*

$$A^T P + PA = -Q, \tag{6}$$

where Q denotes a positive symmetric matrix, and the parameter estimation update law is chosen as

$$\dot{\hat{\theta}} = -\Gamma F(x)^T Pe, \tag{7}$$

where Γ denotes a positive symmetric matrix, then the error system (5) is globally stable. Hence, the response system (2) associated the proposed control law (4) and the parameter estimation update law (7) asymptotically synchronizes the drive system (1).

Proof. Consider a Lyapunov function for the system (5)

$$V = e^T Pe + (\hat{\theta} - \theta)^T \Gamma^{-1} (\hat{\theta} - \theta). \tag{8}$$

Then its first derivative along the error dynamical system (5) is

$$\begin{aligned} \dot{V} &= \dot{e}^T Pe + e^T P\dot{e} + 2(\hat{\theta} - \theta)^T \Gamma^{-1} \dot{\hat{\theta}} \\ &= e^T \left(A^T P + PA\right)e = -2e^T Qe, \end{aligned} \tag{9}$$