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Physics Letters A 344 (2005) 395–400
PHYSICS LETTERS A
www.elsevier.com/locate/pla

Quantum critical scenario from an effective classical renormalization group treatment close to and below four dimensions

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Received 20 April 2005; accepted 10 May 2005

Available online 5 July 2005

Communicated by V.M. Agranovich

Abstract

We suggest a general procedure to analyze quantum criticality for a wide variety of quantum systems of topical interest, close and below four dimensions. The idea is to apply the Wilsonian renormalization group philosophy to an effective classical functional derived from a general quantum action by averaging over degrees of freedom with non-zero Matsubara frequencies. This allows us to describe, in an unified way, all crossovers expected close to a quantum critical point.

PACS: 68.35.Rh; 64.60.Ak

Keywords: Quantum criticality

The physics of quantum phase transitions (QPTs) and the behaviour close to a quantum critical point (QCP) have attracted a great interest in the last decade due to the explosive experimental findings on a wide variety of new materials [1–3].

It has become increasingly clear that the influence of a QCP spreads to the phase diagram well away, so that drastic effects may occur long before this peculiar point is reached. However, due to the considerable experimental and theoretical difficulties in describing QCP effects with reliable precision, the microscopic origin of quantum criticality (QC) remains subject to theoretical debate [3–7].

On the theoretical front, from the work of Millis [8] further studies have shed little additional light on the quantum critical phenomena. So, further investigations are necessary to clarify and extend the limits of existing theories.

The most efficient approach, intensively used to describe QC, is the original Wilsonian renormalization

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2005.05.105

group (RG) framework adapted to path-integral representations. These introduce in the problem the so-called Matsubara time and hence a time-like direction, in addition to the space ones, as an expression of the inextricable link between statics and intrinsic quantum dynamics of the quantum model under study.

This seems an appropriate means to describe QC in clean systems. Nevertheless, when quenched impurity degrees of freedom are taken into account, some conceptual troubles arise [1,9] and the theoretical predictions are no more reliable and clear as for the clean case but appear rather ambiguous and, possibly, misleading [10].

In the present Letter we suggest a new procedure to use, for quantum models, the Wilsonian RG approach which (i) reproduces, in a simple and transparent way, the known quantum critical scenario for clean systems; (ii) allows us to give a unified description of all the involved crossovers, and (iii) may give new insights for overcoming the mentioned difficulties when the randomness is involved. Here we focus on clean models to explore basic ideas and we postpone to a future work the study of the quenched impurity effects on QC.

Let us consider the generalized quantum action [11] (in convenient units)

$$H\{\boldsymbol{\psi}\} = \frac{1}{2}\int d^d x \int_0^{1/T} d\tau \left\{ \left|\nabla \boldsymbol{\psi}(\mathbf{x},\tau)\right|^2 + f\left[\boldsymbol{\psi}(\mathbf{x},\tau)\right] + r_0 \left|\boldsymbol{\psi}(\mathbf{x},\tau)\right|^2 + \frac{u_0}{2}\left|\boldsymbol{\psi}(\mathbf{x},\tau)\right|^4 \right\}, \tag{1}$$

where $\boldsymbol{\psi}(\mathbf{x},\tau)$ is an n-vector real ordering field (with $\boldsymbol{\psi}(\mathbf{x},\tau) = \{\psi^j(\mathbf{x},\tau);\ j = 1,\ldots,n\}$) or a complex one (with $\boldsymbol{\psi}(\mathbf{x},\tau) = \{\psi_1^j(\mathbf{x},\tau) + i\psi_2^j(\mathbf{x},\tau);\ j = 1,\ldots, n/2\}$), T is the temperature and $r_0, u_0, f[\boldsymbol{\psi}(\mathbf{x},\tau)]$ depend on the microscopic quantum model under study [11]. For instance, $f[\boldsymbol{\psi}] = |\partial\boldsymbol{\psi}/\partial\tau|^2$ for transverse Ising-like models [1,11] and $f[\boldsymbol{\psi}] = \boldsymbol{\psi}^* \frac{\partial \boldsymbol{\psi}}{\partial \tau}$ for bosonic models (e.g., Bose gas and transverse XY model [1,11]). In the Fourier space free propagator, $G_0(\mathbf{k},\omega_l) = (r_0 + k^2 + \tilde{f}(\omega_l))^{-1}$, we have $\tilde{f}(\omega_l) = \omega_l^2$ and $\tilde{f}(\omega_l) = -i\omega_l$, for these two cases respectively, where $\omega_l = 2\pi l T$ $(l = 0, \pm 1, \pm 2, \ldots)$ are the Matsubara frequencies.

It is worth noting that the procedure developed below is general and all the results remain formally unchanged both for real and complex ordering fields except for inessential numerical renormalization of the coupling parameter u_0.

With the aim to calculate the partition function, we can start by averaging over the degrees of freedom with $\omega_l \neq 0$. Then, working up to second order in perturbation theory, with $\boldsymbol{\psi}(\mathbf{k},0) \to \boldsymbol{\Phi}(\mathbf{k})$, we obtain a classical $\boldsymbol{\Phi}^4$-Ginsburg–Landau–Wilson (GLW) static functional,[1] with inverse free propagator $\tilde{G}_0^{-1} = \tilde{r}_0 + k^2$ and T-dependent coupling parameters $\tilde{r}_0(T)$ and $\tilde{u}_0(T)$ which are related to the original ones by

$$\tilde{r}_0(T) = r_0 + (n+2)u_0 \frac{T}{V} \sum_{\mathbf{k},\omega_l \neq 0} G_0(\mathbf{k},\omega_l), \tag{2}$$

$$\tilde{u}_0(T) = T u_0 \left[1 - \frac{n+2}{8} u_0 \frac{T}{V} \times \sum_{\mathbf{k},\omega_l \neq 0} G_0(\mathbf{k},\omega_l) G_0(-\mathbf{k},-\omega_l) \right]. \tag{3}$$

Then, we apply to this classical functional the conventional one-loop RG transformation which involves now only the wave-vectors $\mathbf{k}$ with a cut-off $\Lambda = 1$. This yields the well-known equations

$$\frac{d\tilde{r}(l)}{dl} = 2\tilde{r}(l) + (n+2)K_d \frac{\tilde{u}(l)}{1+\tilde{r}(l)}, \tag{4}$$

$$\frac{d\tilde{u}(l)}{dl} = \varepsilon \tilde{u}(l) - (n+8)K_d \frac{\tilde{u}^2(l)}{(1+\tilde{r}(l))^2}, \tag{5}$$

with $\varepsilon = 4 - d$, but to be solved now with the temperature-dependent initial conditions $\tilde{r}(0) = \tilde{r}_0(T)$ and $\tilde{u}(0) = \tilde{u}_0(T)$. One could study these equations directly at $d = 3$ $(\varepsilon = 1)$ following the so-called "direct method" (see, for instance, Refs. [8,12]). However we prefer to work around four dimensions and, only at the end, to extract information about $d = 3$ setting $\varepsilon = 1$ in the final RG results.

At this stage one could analyze the critical properties by standard linearization of Eqs. (4), (5) around the classical Gaussian $(\tilde{r}^* = 0, \tilde{u}^* = 0)$ and non-Gaussian $(\tilde{r}^* = -[(n+2)/2(n+8)]\varepsilon, \tilde{u}^* = [1/K_d(n+$

[1] The same reduction of the quantum degrees of freedom was used by Sachdev [1] to formulate a theoretical approach to finite temperature QC which mixes perturbative predictions and already known $(T = 0)$-RG results, but close to and above the quantum upper critical dimension.