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Domain walls and textured vortices in a two-component Ginzburg–Landau model

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Abstract

We look for domain wall and textured vortex solutions in a two-component Ginzburg–Landau model inspired by two-band superconductivity. The two-dimensional two-component model, with equal coherence lengths and no magnetic field, shows some interesting properties. In the absence of a Josephson type coupling between the two order parameters a "textured vortex" is found by analytical and numerical solution of the Ginzburg–Landau equations. With a Josephson type coupling between the two order parameters we find the system to split up in two domains separated by a domain wall, where the order parameter is depressed to zero.

1. Introduction

The long list of physical systems where topological defects plays an important role includes such seemingly different fields as superconductivity [1], cosmology [2], and singular optics [3], to name a few. In cosmology, e.g., topological defects in the form of vortices have been considered as the seed of galaxy formation by their gravitational field in various different gravitational theories [4]. In singular optics, e.g., optical vortices [5] has been considered in connection with pattern formation in lasers [6]. The study of topological defects in field theoretic models are thus of broad physical interest, though we are here inspired by two-gap superconductivity.

The most famous topological defect in superconductivity is without doubt the Abrikosov vortex, but different types may also be considered. We shall here use a simplified version of the two-component Ginzburg–Landau theory to consider domain walls

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and non-Abrikosov types of vortices. The model is inspired by the discovery of superconductivity in MgB_2 a few years ago [7]. Of particular, interest is the presence of two energy gaps [8] leading to two order parameters in a Ginzburg–Landau type of theory [9]. This kind of multi-component Ginzburg–Landau theory has previously been discussed by several authors [10]. In the present work we consider a simplified version of the two-component Ginzburg–Landau theory, in which magnetic effects are neglected, order parameters are assumed to have equal coherence lengths, and the case of an isotropic superconductor is considered. Even in this simple model, interesting features in the form of topological defects are found.

The Letter is organized in the following way: in Section 2, starting from the free energy functional for the two-component Ginzburg–Landau model we derive analytical solutions corresponding to domain walls and textured vortices. In Section 3 we solve the coupled partial differential equations numerically and discuss the relationship to the analytical results obtained in Section 2. Section 4 contains our conclusion.

2. Theory

We consider the two-dimensional static case described by the free energy functional

$$F = \int d^2x \left(\frac{1}{2}(\nabla\psi_1)(\nabla\psi_1)^* + \frac{1}{2}(\nabla\psi_2)(\nabla\psi_2)^* + V(|\psi_1|, |\psi_2|) - \eta(\psi_1^*\psi_2 + \psi_1\psi_2^*) \right), \tag{1}$$

where ψ_1 and ψ_2 are the two normalized order parameters, η is the strength of the Josephson type coupling between the order parameters, and V is the standard 4th order Ginzburg–Landau potential in each of the order parameters

$$V(|\psi_1|, |\psi_2|) = -\frac{1}{2}(|\psi_1|^2 + |\psi_2|^2) + \frac{1}{4}(|\psi_1|^4 + |\psi_2|^4), \tag{2}$$

where we have assumed equal coherence lengths for the two order parameters.

2.1. Domain walls

We first consider a domain wall linking two phases. Assuming real order parameters and $\psi_1 = \pm\psi_2$, the equations

$$\nabla^2\psi_1 + (1 \pm 2\eta)\psi_1 - \psi_1^3 = 0 \tag{3}$$

is obtained from the Euler–Lagrange equations. Considering the case where ψ_1 is independent of the y-coordinate, a solution to the above equation becomes

$$\psi_1 = \sqrt{1 \pm 2\eta}\tanh\left(\sqrt{\frac{(1 \pm 2\eta)}{2}}x\right), \tag{4}$$

describing a domain wall located along the y axis. Note, for $\eta > 0$ ($\eta < 0$) the solution with $\psi_1 = \psi_2$ ($\psi_1 = -\psi_2$) has a lower energy than the solution with $\psi_1 = -\psi_2$ ($\psi_1 = \psi_2$).

Note that the domain wall is similar to a SNS Josephson π-junction [13].

2.2. Textured vortices

We now consider textured vortex solutions. Assuming real order parameters, we may write

$$\psi_1 = F(x, y)\cos\phi(x, y),$$
$$\psi_2 = F(x, y)\sin\phi(x, y), \tag{5}$$

where F and ϕ denote amplitude and phase, respectively. The field equations then become

$$\nabla \cdot (F^2\nabla\phi) + 2\eta F^2\cos 2\phi + \frac{F^4}{4}\sin 4\phi = 0 \tag{6}$$

and

$$\nabla^2 F - F(\nabla\phi)^2 = \frac{F^3}{4}(3 + \cos 4\phi) - F(1 + 2\eta\sin 2\phi). \tag{7}$$

An approximate analytical solution to Eqs. (6) and (7) in the far-field is obtained by assuming vanishing derivatives of F in Eq. (7), leading to $F = 0$ or

$$F^2 = \frac{4(1 + 2\eta\sin 2\phi - (\nabla\phi)^2)}{3 + \cos 4\phi}. \tag{8}$$

Inserting the latter expression into Eq. (6) and neglecting ∇F, we obtain the approximative equation

$$\nabla^2\phi + 4\eta\cos 2\phi + \frac{1}{2}\sin 4\phi = 0, \tag{9}$$