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Effects of long-range magnetic interactions on DLA aggregation

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Abstract

An extra degree of freedom is introduced in the well-known diffusion-limited aggregation model, i.e., the growth entities are "spin" taking. The model with long-range magnetic interactions that decay as $\beta C/r^{\alpha}$ on two-dimensional square lattices is studied for different values of α. This model leads to a wide variety of kinetic processes and morphology distribution with both the coupling energy βC and the range of the interactions, i.e., the exponent α. The simulated result of the model shows that the "quenching" of the degree of freedom on the cluster by the long-range magnetic interactions leads to branching or compactness, but, moreover, to combined geometric and physical "transitions" of the aggregations with the growth parameters.

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1. Introduction

For the past 60 years, kinetic growth models have received a great deal of attention because of the set of universality classes in which they belong and the natural processes which they generate [1]. Growth models are studied in many domains of science such as gelation [1], percolation [2], crystal growth [3], fracture [4], sedimentation [5], or dielectric breakdown [6]. Among the models, diffusion-limited aggregation (DLA) [7] model is very applicable, which was introduced in 1981 by Witten and Sander. It generates aggregation of particles in a cluster through a Brownian motion. The DLA model provides a basis for understanding a large number of natural formation phenomena [7–12]. However, many questions about this kinetic growth model still remains open, for instance, the theoretical "physical" understanding of the DLA growth is a big challenge for the future.

Usually, natural systems are constituted by entities taking different states. E.g., copolymers are macro-

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molecules mode of two kinds of monomers [13]. Some bacterial cells like salmonella have some gene taking two states (“on” and “off”) [14]. It is, thus, of interest to generalize the kinetic growth models in order to study the impact of a competition between physically different entities on the kinetic processes.

Microscopic pair interactions that decay slowly with the distance r between particles appear in different physical systems. Typical examples are gravitational and Coulomb interactions, where the potential decays as $1/r$. Several other important examples can be found in condensed matter, such as dipolar (both electric and magnetic) and Ruderman–Kittel–Kasuya–Yosida (RKKY) interactions, both are proportional to $1/r^3$. Effective interactions with a power-law decay $1/r^\alpha$ [15–20], with some exponent $\alpha \geqslant 0$, appear also in other related problems such as critical phenomena in highly ionic systems [21], Casimir forces between inert uncharged particles immersed in a fluid near the critical point [22], and phase segregation in model alloys [23].

The model presented here is quite different. Specifically, we introduce in the DLA model an internal degree of freedom, i.e., a spin taking two states with long-range classical Ising energy. We will show that the “quenching” of the degree of freedom on the cluster by the long-range magnetic interactions leads to branching or compactness but moreover to combined geometric and physical “transitions” of our simulated aggregations with the growth parameters.

2. Model description

The magnetic diffusion-limited aggregation (MDLA) model with long-range interactions that decay as $1/r^\alpha$ is based on the ordinary DLA model and the aggregation of spins moving toward a cluster through a characteristic motion is introduced. On a two-dimensional square lattice with lattice spacing a, the growth rule is defined by the following steps:

(i) An initial spin S_0 (up or down) is dropped on a “seed site”. The extension of this monoparticle cluster is $r_{\max} = a$.

(ii) A diffusion spin or particle, up or down, is dropped onto a circle of $r_{\max} + 20a$, centered on the seed site.

(iii) A choice is then made for both the next site and the next state orientation of the diffusing spin, as the spin is allowed to flip or not flip. The probabilities of jumping to one of the four neighbor sites are defined as proportional to $\exp(-\Delta\beta E)$, where $\Delta\beta E$ is the local gain of the dimensionless Ising energy with long-range interactions between the initial and the final states defined by

$$\beta E = -\frac{\beta}{2}\sum_{\langle i,j\rangle} J(r_{ij}) S_i S_j \tag{1}$$

with

$$J(r_{ij}) = \frac{C}{r_{ij}^\alpha} \quad (\alpha \geqslant 0), \tag{2}$$

(where r_{ij} is the distance in lattice units) between particle i and j, and where the sum $\sum_{\langle i,j\rangle}$ in Eq. (1) runs over all distinct pairs of particles. C is considered to be an exchange integral, while β is a parameter such that βC is dimensionless, and α corresponds to the distance dependence of the exchange energy. The probabilities of the eight possible configurations for each jump are renormalized and one specific configuration is chosen through a random number generator.

(iv) If the spin moves outside the circle of radius $r_{\max} + 30a$ centered on the seed site, the spin is removed from the process and one returns to step (ii). If the spin jumps onto a perimeter site, i.e., an empty site connected to a spin of the cluster, it sticks immediately on the cluster and the next diffusing spin is launched (i.e., return to step (ii)). Then the value of $r_{\max}$ of the cluster is adapted to the largest distance between the farthest cluster site and the seed site. However, if the spin jumps to any other unconnected site, a new step (iii) is attempted. After sticking to the cluster, the orientation of the new attached particle is not changed any more.

(v) The launching and diffusing procedures are repeated until a desired number N of spins frozen on the cluster is reached.

One should note that for $\beta C = 0$, the spins are decoupled and the model reduces to the ordinary DLA model [7]. By varying α we can modulate the range of the interactions, and βC can be interpreted as a dimensionless temperature (like K in Ref. [24]).