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Superconducting pairing from repulsion: Contact potential approximation

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Abstract

Pairing of repulsive particles with large total momentum results in essential relative-motion momentum dependence of the superconducting order parameter with a nodal line crossing the Fermi contour inside kinematically accessible domains of the momentum space. Repulsive interaction potential resulting in the superconducting pairing is reduced to a stepwise one with a degenerate kernel having two eigenvalues of opposite sign. Using such a kind of contact potential approach the self-consistency equation is transformed into a system of two integral equations determining average values of the order parameter inside the subdomains of its constant sign. The solution of this equation system is obtained in the weak coupling limit.

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High-temperature superconducting (HTSC) cuprates, unlike conventional superconductors, exhibit highly unusual properties both in normal and superconducting (SC) states [1] and there is a reason to think that the superconductivity of these compounds cannot be treated completely in the framework of the mean-field theory by Bardeen, Cooper and Schrieffer corresponding to singlet pairing of particles with zero total momentum due to phonon mediated attraction inside a narrow layer of the momentum space enclosing the Fermi surface. We suppose that, due to a proximity of the antiferromagnetic and SC phases, the singlet pairing with large total momentum from repulsive interaction may dominate the superconductivity of HTSC cuprates [2]. The SC order from repulsion arises in the mean-field scheme if and only if the repulsive potential has at least one *negative* eigenvalue [3]. Under the condition of perfect *mirror nesting* [2], a non-trivial solution of the self-consistency equation exists even if the interaction is arbitrarily weak.

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Perfect mirror nesting results from highly special electron dispersion when some pieces of the Fermi contour (FC) of a quasi-two-dimensional electron system of cuprate compound coincide with pieces of relative-motion kinetic energy isoline corresponding to a pair with given total momentum [2]. It is naturally to consider these pieces as *"pair" Fermi contour* (PFC) which plays the same role in the relative motion of the pair as the FC plays in the motion of electrons and holes.

In the case of repulsive interaction, the order parameter $\Delta(\boldsymbol{k})$ arising as a result of SC pairing with large total momentum depends on the relative-motion momentum $\boldsymbol{k}$. One can see that $\Delta(\boldsymbol{k})$ has to be a function with alternating signs inside the *kinematically accessible domain* Ξ_K of the momentum space [2]. The form and size of such a domain are determined by the total pair momentum $\boldsymbol{K}$ and the FC geometry. The function $\Delta(\boldsymbol{k})$ has a *nodal line* crossing the PFC at some points inside each of the crystal equivalent domains Ξ_K. The order parameter symmetry is determined by the interaction mixing states of pairs belonging to equivalent domains and may be of either d-wave, when the order parameter changes its sign under $\pi/4$ rotation, or extended s-wave symmetry (or else, $s+g$ symmetry [4]) in the opposite case.

The nodal line divides the kinematically accessible domain Ξ (here and latter on, the subscript K is omitted) into two sections, Ξ_1 and Ξ_2, both containing a part of the PFC, as it is shown schematically in Fig. 1. If one defines an average value Δ_p of the order parameter over the section Ξ_p where $p = 1, 2$ the nonlinear integral self-consistency equation which determines the $\boldsymbol{k}$-dependent order parameter can be transformed into a system of two integral equations introduced in [2] phenomenologically. The solution of this system turns out to be a piecewise constant function with opposite-sign components Δ_1 and Δ_2. To describe scattering of particles both inside and between the filled and vacant parts of the domain Ξ one needs to introduce three phenomenological constants relating to the areas of the momentum space accessible for scattering [2]. In the case of pairing with large total momentum such constants can be expressed by means the interaction potential parameters in the framework of supposed here approach being a development of the well-known contact potential approximation [5].

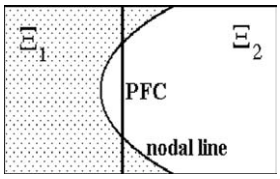

Fig. 1. Kinematically accessible domain of the momentum space (the domain of definition of relative-motion momentum of pairs with large total momentum, schematically).

Indeed, because of smallness of the kinematically accessible domain, one can express the interaction matrix element $U(\boldsymbol{k}-\boldsymbol{k}')$ in a simple form [3]

$$U_d(\boldsymbol{k}-\boldsymbol{k}') = U_0 r_0^2\bigl[1-(\boldsymbol{k}-\boldsymbol{k}')^2 r_0^2/2\bigr] \tag{1}$$

which may be considered as the first two terms of the corresponding power series converging inside Ξ. Here, r_0 is a characteristic interaction length whereas $U_0 r_0^2 \equiv w_0$ has the meaning of an interaction strength. A passage to the limit $r_0 \to 0$ under the condition that $w_0 = \text{const}$, typical for the contact potential approach, leads to a possibility to reduce the degenerate kernel (1) into a piecewise constant (inside Ξ) kernel defined by three parameters corresponding to scattering both inside and between the sections Ξ_1 and Ξ_2 of the domain Ξ.

In this Letter, we present an approach which allows one to obtain these parameters. Using such a contact potential approximation, we present, in the weak-coupling limiting case, a solution of the self-consistency equation.

Cooper's solution of the problem relating to two attracting particles exciting above (or under) the Fermi surface leads to a conclusion that the ground state of normal Fermi liquid should be unstable with respect to association of the particles into pairs with zero total momentum. One can obtain a similar result also in the case of pairing of repulsing particles with non-zero total momentum [6].

A wave function of the relative motion of the pair is determined by the equation

$$\psi(\boldsymbol{k}) = G(\boldsymbol{k}; E)\int\limits_{\Xi} U(\boldsymbol{k}-\boldsymbol{k}')\psi(\boldsymbol{k}')\,d^2k' \tag{2}$$

which has a non-trivial solution when the energy of the relative motion of the pair $E < 0$. The integral has to be taken over the domain Ξ. Green function corresponding to free relative motion of the pair is defined