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Adaptive chattering free variable structure control for a class of chaotic systems with unknown bounded uncertainties ☆

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Abstract

A new adaptive control scheme is developed for a class of chaotic systems with unknown bounded uncertainties. It is implemented by using variable structure control. The concept of extended systems is used such that continuous control input is obtained to avoid chattering phenomenon as frequently in the conventional variable structure systems. Furthermore, it is worthy of note that the proposed adaptive control scheme does not involve any information about the bounds of uncertainties. Thus, the limitation of knowing the bounds of uncertainties in advance is certainly released. A numerical simulation is included to verify the validity of the developed adaptive chattering free variable structure control.

Keywords: Chaotic system; Adaptive control; Variable structure control; Extended systems; Chattering

1. Introduction

Nowadays more and more chaotic phenomena are being found in many engineering systems. Chaotic system is a very complex dynamical nonlinear system and its response possesses some characteristics, such as excessive sensitivity to initial conditions, broad spectra of Fourier transform, and fractal properties of the motion in phase space [1]. Due to its powerful applications in chemical reactions, power converters, biological systems, information processing, secure communications, etc., controlling these complex chaotic dynamics for engineering applications has emerged as a new and attractive field and has developed many profound theories and methodologies to date. In 1990, Ott et al. [2] showed that a chaotic attractor could be converted to any one of a large number of possible attracting time-periodic motions by making only small time-dependent parameter per-

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turbations. Chen and Dong [3] showed that the conventional linear state feedback control is still an effective tool for chaotic systems. Vincent and Yu [4] used a bang–bang controller to cope with the control problem of chaotic system. In addition, several control methods have also been successfully applied to chaotic systems, e.g., robust control [5], adaptive control [6–9], H^{∞} control [10], digital redesign control [11], etc. In the above-mentioned studies, the controllers were synthesized based on the full knowledge of the chaotic model and the upper-bounds of uncertainties and disturbance. However, in real physical systems or experimental situations, this may not be easy obtained in practice and the control schemes obtained using these upper-bounds of uncertainties and disturbances yields over-conservative (high) feedback gains [12]. At present, little attention has been given to adaptive controlling uncertain chaotic systems without the limitation of knowing the bounds of uncertainties.

For designing a robust control, variable structure control (VSC) is frequently adopted due to its inherent advantages of easy realization, fast response, good transient performance and insensitive to variation in plant parameters or external disturbances. Recently, Tsai et al. [13], Yang et al. [14], Yau et al. [1] have successfully applied the concept of variable structure control to cope with the control problem for uncertain chaotic systems. However, in the above works, the control schemes are all still under the limitation of knowing the bounds of uncertainties and chattering of the control input occurred in Tsai et al. [13]. For the above reasons, it is highly desirable to propose a new adaptive chattering free controller for chaotic systems to not only preserve the advantages of variable structure control but also release the limitation of knowing the bounds of uncertainties.

In this Letter, the tracking problem for uncertain chaotic systems with unknown bounded uncertainties is considered. An adaptive variable structure control (VSC) is designed to guarantee the existence of the sliding mode for the tracking error dynamics. In particular, the chaotic systems can be driven to arbitrary trajectory, even when the desired trajectories are not located on the embedded orbits of a chaotic system. Since the continuous control input is used in this Letter, chattering phenomenon is removed. Meanwhile, the limitation of knowing the bounds of uncertainties is also removed due to the adaptive mechanism. Finally, a numerical example is illustrated to demonstrate the validity of the derived adaptive VSC. Throughout this Letter, it is noted that $\lambda(W)$ denotes an eigenvalue of W and $\lambda_{\max}(W)$ represents the $\max[\lambda_i(W)]$, $i = 1, \dots, n$. $|w|$ represents the absolute value of w and $\|W\|$ represents the Euclidean norm when W is a vector or the induced norm when W is a matrix. $\operatorname{sign}(s)$ is the sign function of s, if $s > 0$, $\operatorname{sign}(s) = 1$; if $s = 0$, $\operatorname{sign}(s) = 0$; if $s < 0$, $\operatorname{sign}(s) = -1$. I_n represents the identity matrix of $n \times n$.

2. System definition for uncertain chaotic systems

In this Letter, we consider a class of uncertain chaotic systems with unknown bounded uncertainties described as

$$\begin{aligned}
&\dot{x}_i(t) = x_{i+1}(t), \quad i = 1, \dots, n-1, \\
&\dot{x}_n(t) = f(X, t) + \Delta f(X, t) + \delta(t) + u(t),
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
X(t) &= \left[x_1(t), x_2(t), \dots, x_n(t)\right] \\
&= \left[x(t), \dot{x}(t), \dots, x^{(n-1)}(t)\right] \in R^n
\end{aligned}$$

is the state vector, $f(X, t) \in R$ is a given nonlinear function of X and t, $u(t) \in R$ is the control input, $\Delta f(X, t)$ is the unknown parameter uncertainty applied to the system, and $\delta(t)$ denotes the unknown external disturbance. The superscript n denotes the order of differentiation. In many previous reports, the upper-bounds of parameter uncertainties and external disturbance must be obtained in advance. However, in this study, this limitation of knowing the upper-bounds of uncertainties will be released.

Remark 1. (1) is not restrictive. Several nonlinear chaotic systems can be transformed into the controllable canonical form (1) with some state transformation [15,16]. For example, Rössler systems [15], Lur'e-like system [16] and Duffing–Holmes system [22] all belong to the class defined by (1).

The control problem is to get the system to track an n-dimensional desired vector $X_d(t)$ (i.e., the original nth-order tracking problem of state $x_d(t)$ as discussed