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# Positive periodic solutions of delayed periodic Lotka–Volterra systems ☆

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**Abstract**

In this Letter, for a general class of delayed periodic Lotka–Volterra systems, we prove some new results on the existence of positive periodic solutions by Schauder's fixed point theorem. The global asymptotical stability of positive periodic solutions is discussed further, and conditions for exponential convergence are given. The conditions we obtained are weaker than the previously known ones and can be easily reduced to several special cases.


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*Keywords:* Lotka–Volterra system; Periodic dynamical system; Schauder's fixed point theorem; Global exponential stability; Lyapunov function

## 1. Introduction

The Lotka–Volterra model was first proposed to describe the predator–prey relationship in an ecosystem, and soon became well known and formed the basis of many important models in mathematical biology and population dynamics [13,26]. In recent years, it has also been found with successful and interesting applications in physics, chemistry, economics and other fields, for example, see [4,7,8,15,16,18,25]. The Lotka–Volterra type neural networks, which are derived from conventional membrane dynamics of competing neurons, provide a mathematical basis for understanding neural selection mechanisms [6]. This type of neural networks have been implemented by subthreshold MOS integrated circuits [1]. In [14], it was shown that the continuous-time recurrent neural networks

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can be embedded into Lotka–Volterra models by changing coordinates, which suggests that the existing techniques in the analysis of Lotka–Volterra systems can also be applied to recurrent neural networks.

Due to its importance in real applications, the analysis of dynamic behaviors, including stability, periodic oscillation, chaos and bifurcation, plays a key role in the studies on these dynamical models. For references to earlier literature, see [9,10,19] and the cites therein. Since the phenomena of periodicity are very common in real world, which may be caused by seasonality of environments, fluctuation of markets or oscillation of physical variables, the dynamic properties of periodic dynamical systems have attracted much attention these years. In particular, motivated by the biological background, there has been great interest discussing the positive periodic solutions of Lotka–Volterra systems, which implies the permanence of all species in an ecosystem. There has been an extensive literature addressing these issues, for references, see [5,11,12,20,22–24,27].

There are different ways to analyze the positive periodic solutions of periodic Lotka–Volterra systems. Some researchers discussed the persistence or permanence of the systems first and proved the existence and stability of positive periodic solutions after that [22,23]. In fact, the persistence or permanence of the systems is not necessary for the existence of positive periodic solutions, which leads to strict conditions. In some literature, various techniques such as the continuation theorem of coincidence degree theory and the theory of monotone semiflow generated by functional differential equations, were used to obtain some existence results [11,12,17,20]. However, the applications of complicated theories made the proofs difficult and the conditions not weak enough.

In this Letter, we adopt a direct method to address the problem. First, we prove the existence of positive periodic solutions directly by Schauder's fixed point theorem, which was proposed in [21], without introducing the concepts of persistence and permanence. We further discuss the stability of positive periodic solutions and obtain some results involving not only the global asymptotical stability but also the global exponential stability which was seldom referred to before. The model we consider here is a very general form of Lotka–Volterra systems, including cooperative systems, competitive systems and their hybrids, and also including time-varying delays and distributed delays. Comparisons show that our conditions are weaker than the previously known ones, and can be easily reduced to special cases of cooperative systems and competitive systems.

This Letter is organized as follows. In the next section, model description and some preliminaries are given. In Section 3, we prove some theorems on the existence of positive periodic solutions, and compare them with several existing results. Global asymptotical stability and global exponential stability are addressed in Section 4. A numerical example is given in Section 5, and finally, conclusions are drawn in Section 6.

## 2. Model description and preliminaries

In this Letter, we investigate the general form of delayed periodic Lotka–Volterra systems described by the following delayed differential equations:

$$\frac{dx_i(t)}{dt} = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t) - \sum_{j=1}^{n}\int_{-\infty}^{0} x_j(t+s)\,d_s\mu_{ij}(t,s)\right], \quad i = 1, 2, \ldots, n, \tag{1}$$

where $b_i(t)$, $a_{ij}(t)$ are continuous periodic functions with period $\omega > 0$, $\int_0^{\omega} b_i(u)\,du > 0$, $a_{ii}(t) > 0$, and for any fixed $t \geqslant 0$, $d_s\mu_{ij}(t,s)$ are generalized measures such that $d_s\mu_{ij}(t+\omega,s) = d_s\mu_{ij}(t,s)$ for all $i, j = 1, 2, \ldots, n$. The initial condition is

$$x_i(s) = \phi_i(s) \quad \text{for } s \in (-\infty, 0], \tag{2}$$

where $\phi_i(s)$ is bounded continuous functions on $(-\infty, 0]$ and $\phi_i(s) > 0$, for all $i = 1, 2, \ldots, n$.

Let $d_s\mu_{ij}(t,s) = b_{ij}(t)\delta(s + \tau_{ij}(t))\,ds + c_{ij}(t,s)\,ds$, where $\delta(\cdot)$ is the Dirac delta function, $b_{ij}(t)$, $\tau_{ij}(t)$, $c_{ij}(t,s)$ are continuous $\omega$-periodic functions with respect to $t$, $\tau_{ij}(t) \geqslant 0$, for all $i, j = 1, 2, \ldots, n$, then system (1)