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Fuzzy adaptive synchronization of uncertain chaotic systems

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Abstract

This Letter presents an adaptive approach for synchronization of Takagi–Sugeno (T–S) fuzzy chaotic systems. Since the parameters of chaotic system are assumed unknown, the adaptive law is derived to estimate the unknown parameters and its stability is guaranteed by Lyapunov stability theory. The control law to be designed consists of two parts: one part that can stabilize the synchronization error dynamics and the other part that estimates the unknown parameters. Numerical examples are given to demonstrate the validity of the proposed adaptive synchronization approach.

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Keywords: T–S fuzzy model; Chaos synchronization; Adaptive control; Unknown parameter estimation

1. Introduction

It is well known that since the pioneering work of Carroll and Pecora [1], synchronization of two chaotic dynamical systems is one of the most important applications of chaos and has been paid much attention. In some engineering applications such as secure communications, chaotic synchronization has been investigated extensively and found valuable since chaotic signals are typically broadband, noiselike, difficult to predict, and can hide information efficiently and securely [2]. To recover the message the receiver should synchronize with the transmitter in secure communications. Therefore, the problem of chaotic synchronization should be solved. Many approaches for synchronization of chaotic systems based on exact chaotic model have been proposed. In [3,4], the feedback control techniques to achieve chaotic synchronization were reported. However, some parameters of chaotic system

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cannot be exactly known in priori. If chaotic system is partly known, that is, its differential equation is known but some or all of the parameters are unknown, the synchronization scheme based on exact model of chaotic systems may be infeasible. To solve the problem of chaotic synchronization with unknown parameters, adaptive strategy for a variety of chaotic systems has been presented [5–7].

In recent years, fuzzy logic has received much attention from the control theorists as a powerful tool for the nonlinear control. Among various kinds of fuzzy methods, Takagi–Sugeno fuzzy system is widely accepted as a tool for design and analysis of fuzzy control systems [8]. T–S fuzzy model is frequently used for mathematical simplicity of analysis. In this type of fuzzy model, local dynamics in different state space regions are represented by linear models. The overall model of the system is achieved by fuzzy "blending" of these linear models. The T–S fuzzy model can express a highly nonlinear functional relation with a small number of implications of rules [9]. T–S fuzzy control theory has been applied to chaos control and synchronization in some literatures [10–12]. In [10], the authors proposed chaotic synchronization and chaotic model following control by applying a fuzzy feedback control law. The authors in [11,12] proposed fuzzy observer-based chaotic synchronization and secure communication. Recently, the authors in [13] have reported an adaptive parameter estimator based on T–S fuzzy models.

In this Letter, we propose a fuzzy model-based adaptive approach for synchronization of chaotic systems with unknown parameters. T–S fuzzy model is adopted for the modeling of chaotic drive and response systems. Based on this fuzzy model, synchronization error dynamics is derived, fuzzy controller is designed and its stability is analyzed. We use state error between the state of the drive system and the state of the response system to construct control law in response system which achieves robust synchronization. To estimate the unknown parameters of the drive system, the adaptive law is also derived by using Lyapunov stability theory and used as a part of the controller in the response system.

The rest of this Letter is organized as follows. In Section 2, some preliminaries for T–S model of chaotic system and problem formulation are given. Fuzzy model-based adaptive synchronization, that is, the design of the fuzzy controller which achieves robust synchronization and adaptive law to estimate unknown parameters are also investigated in this section. The validity and the effectiveness of the proposed adaptive approach are examined through numerical examples, Duffing oscillator and Lorenz attractor, in Section 3. Finally, some conclusions are presented in Section 4.

2. T–S fuzzy model-based adaptive synchronization

In system analysis and design, it is important to select an appropriate model representing a real system. As a expression model of a real plant, we use the fuzzy implications and the fuzzy reasoning method suggested by Takagi and Sugeno [9]. The T–S fuzzy system can represent a general class of nonlinear system. For synchronization problem, the drive system and the response system are designed such that internal states of the drive system and response system are to be same. Let us represent chaotic drive system as follows.

Drive system rule i:
If $z_1(t)$ is $M_{i1}, \ldots, z_p(t)$ is M_{ip}. Then

$$\dot{x}(t) = A_i x(t) + A_{ui} x(t) + B_{ui} \zeta(t), \quad i = 1, \ldots, r, \tag{1}$$

where $x \in R^n$ is the state of the drive system, $z(t) = [z_1(t) \ldots z_p(t)]^T$ is a vector of the premise variables, M_{ij} $(j = 1, \ldots, p)$ are fuzzy sets, r is the number of fuzzy rules, A_i, A_{ui} and B_{ui} are some constant matrices of compatible dimensions, A_{ui} and B_{ui} are the unknown parameters of the drive system, $\zeta(t) \in R^m$ may denote the oscillated force or constant value in the chaotic dynamics.