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Asymmetry-induced spectral peaks in periodically forced noisy bistable systems

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Abstract

The occurrence of finite-width peaks in the spectral background of a time-sinusoidally forced overdamped bistable Duffing oscillator, subject to a noise-floor, is reported. The peaks, which only occur in the presence of a symmetry-breaking dc signal, are located at approximately the odd integer multiples of the forcing frequency. An analytic expression for the height of the peaks is derived and validated via computer simulations. The peak heights display a non-monotonic dependence on the system asymmetry.
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1. Introduction

The overdamped, bistable, and periodically forced Duffing system, in the presence of noise, has been the subject of numerous studies of activation phenomena, amongst which "stochastic resonance" (SR) [1] has garnered much recent interest. Among the systems for which this model affords a good qualitative description of the dynamics, we mention the bistable ring laser, electron paramagnetic resonance, magnetic field sensors, chemical reactions, as well as neuronal and sociological systems. Despite these studies, however, the Duffing system still poses a significant theoretical challenge, mainly due to the richness of noise-mediated cooperative phenomena that occur in the presence of the non-stationarity stemming from the periodic force.

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The power spectral density (PSD) of this system can be described, approximately, via a two-state approximation [2–4]. For the symmetric system the main features of the PSD are well known. In the (linear response) regime of strong noise and weak periodic forcing, the PSD has a Lorentzian background with a single delta function peak superposed at the forcing frequency [2]. With increasing signal amplitude, one observes delta peaks at the odd multiples of the forcing frequency. In the opposite regime of weak noise and strong (but still subthreshold) periodic forcing—the strongly non-linear regime—the PSD takes on new features. In particular, dips occur at even multiples of the forcing frequency and delta functions occur at odd integer multiples. The dips in the spectral background were observed experimentally [5–7], predicted theoretically in their singular form [8], and described by a phenomenological law [7]. Analytic expressions describing the dips were first derived in [4].

In practice, however, physical systems are usually asymmetric. Hence, new phenomena can occur, e.g., the non-zero currents induced in stochastic ratchets [9] and the observation of even and odd spectral harmonics in the PSD of a bistable system [5,10,11]. Here, we present a comprehensive theoretical analysis of the PSD of the asymmetric Duffing system in the limit of weak noise and strong subthreshold forcing, the strongly nonlinear regime. Recent work [12] has indicated that this is the optimal working regime for signal detection applications. We report a previously undiscovered feature of the PSD, specifically, the occurrence of finite-width spectral peaks at (approximately) odd integer multiples of the forcing frequency. These peaks are entirely the result of the asymmetry. We stress that these peaks occur in the continuous background of the spectrum and should not be confused with the spectral harmonics (delta functions) that also occur at integer multiples of the forcing frequency.

In [13] we found the distributions of switch times for the asymmetric bistable system driven by Gaussian noise $\xi(t)$ with zero mean $\langle\xi(t)\rangle = 0$ and correlation function $\langle\xi(t)\xi(t')\rangle = 2D\delta(t-t')$, and the strong (but *subthreshold*) periodic force $A\cos\Omega t$,

$$\dot{x} = \frac{\partial V(x,t)}{\partial x} + \xi(t), \tag{1}$$

where the (time-dependent) double-well potential is $V(x,t) = -(a/2)x^2 + (b/4)x^4 + cx - Ax\cos(\Omega t)$. We obtained results under the following approximations: (1) adiabatic dynamics ($\Omega^{-1} \gg \tau_{\mathrm{rel}}$, the well-relaxation time), and (2) weak noise intensity and strong (but *subthreshold*) sinusoidal forcing, $A/D \gg 1$. The system asymmetry enters through the (non-zero) parameter c; in a real system, the asymmetry could stem from a non-zero noise mean value, or a constant component in the external forcing term, for example.

2. Calculation of the power spectrum

The calculation follows a method that was first described in [14] and adapted to the calculation of the PSD of a periodically driven symmetric bistable system in [4]. To start, it is convenient to replace the full dynamics (1) by the two state approximation,

$$y = \begin{cases} 2X\colon\ x > 0, \\ 0\colon\ x < 0, \end{cases} \tag{2}$$

where we tacitly neglect intrawell motion. Then, the switching dynamics can be described by the master-equation,

$$\dot{w}_1 = -\big(W_{12}(t) + W_{21}(t)\big)w_1 + W_{21}(t), \qquad w_1 + w_2 = 1, \tag{3}$$

where w_1, w_2 are the probabilities of being in the states 1 ($y = 0$) and 2 ($y = 2X$), and $W_{12}(t)$, $W_{21}(t)$ are the transition rates from the states $1 \to 2$ and $2 \to 1$, respectively. These rates are periodic with period $T = 2\pi/\Omega$, and can be approximated [13], for weak noise, by Gaussian distributions having widths $\delta t_1 = \sqrt{D/|x_1(nT) - x_s(nT)|A\Omega^2}$ and $\delta t_2 = \sqrt{D/|x_2((n+1/2)T) - x_s((n+1/2)T)|A\Omega^2}$, and heights $W_{12\,\max}$ and $W_{21\,\max}$, n being an integer. Here x_1 and x_2 are the locations of the minima of the potential, and x_s, the location of its maximum. The Gaussians describing $W_{12}(t)$ and $W_{21}(t)$ are therefore located at integer multiples of $T/2$. However, the finite width

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