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Variable separation solutions and interacting waves of a coupled system of the modified KdV and potential BLMP equations

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ABSTRACT

The multilinear variable separation approach (MLVSA) is applied to a coupled modified Korteweg–de Vries and potential Boiti–Leon–Manna–Pempinelli equations, as a result, the potential fields u_y and v_y are exactly the universal quantity applicable to all multilinear variable separable systems. The generalized MLVSA is also applied, and it is found that u_y (v_y) is rightly the subtraction (addition) of two universal quantities with different parameters. Then interactions between periodic waves are discussed, for instance, the elastic interaction between two semi-periodic waves and non-elastic interaction between two periodic instantons. An attractive phenomenon is observed that a dromion moves along a semi-periodic wave.

1. Introduction

Nonlinear partial differential equations (NPDEs) are essentially used to describe various nonlinear phenomena in the nature [1–3]. Accordingly, solutions, either analytic or numeric, play a vital role in the explanations of these attractive phenomena. Commonly, it is rather difficult to obtain analytical solutions of NPDEs. However, fortunately, a class of NPDEs known as integrable systems or soliton equations, possess exact solutions, especially soliton solutions. Solitons have wide applications in many disciplines, such as optical solitons in nonlinear fibers and photonic crystals [4], polaritonic solitons in a Bose–Einstein condensate trapped in a soft optical lattice [5], electrostatic solitons in pair plasmas [6], solitons in the lambda-repressor protein [7], and so on.

Many effective methods have been developed to obtain soliton solutions of integrable NPDEs. The systematic methods include, just name a few, the inverse scattering transform (IST) [8], Hirota bilinear method [9], Bäcklund and Darboux transformations [10], truncated Painlevé expansion approach [11], etc. It is remarkable that the IST can be interpreted as a nonlinear extension of the Fourier transform applicable only for linear differential equations. While the method of separation of variables, the other powerful method for solving linear differential equations, has several extensions, such as the nonlinearization of Lax system [12], the symmetry constraint method [13], the generalized conditional symmetry approach for functional and derivative-dependent functional separable solutions [14], the multilinear variable separation approach (MLVSA) [15,16] and the generalized MLVSA (GMLVSA) [17], and so on.

In this Letter, we focus on the applications of the MLVSA and GMLVSA, which have been reviewed in [18]. It is known that both methods can be applied to obtain nonlinear variable separation solutions of NPDEs, either integrable or non-integrable [19]. Besides, it is featured that solutions from both methods include functions with variables really separated, and solutions of integrable systems from MLVSA can have at least one while at most two arbitrary functions, but those from GMLVSA can have more than two arbitrary functions. Though they are proposed for $(2+1)$-dimensional systems, they can also be used to solve $(1+1)$-dimensional [20] and $(3+1)$-dimensional nonlinear systems [21]. The difficulty usually lies in the second step that is to derive the variable separated equations and then to solve them. If it is succeeded, then one may study abundant nonlinear localized excitations by making advantage of the arbitrary functions. Recently, a new direct variable separation algorithm based on the MLVSA has been proposed and applied to the $(2+1)$-dimensional mKdV equation and

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the $(3+1)$-dimensional BKP equation [22]. The similar variable separation solutions can also be obtained by other methods such as the extended tanh-function method (ETM), the improved ETM, the projective Riccati equation method (PREM) and the extended PREM. It has been reported that the seemingly different variable separation solutions obtained by means of the MLVSA, the extended PREM and the improved ETM are equivalent to each other [23].

In the following, we utilize the MLVSA and GMLVSA to search for variable separation solutions of a coupled system

$$u_t + u_{xxx} + 3u^2 u_x + 3(uv_x)_x = 0, \tag{1a}$$

$$v_{yt} + v_{xxxy} + 3\left(v_x v_y + u^2 v_y + uu_{xy}\right)_x = 0. \tag{1b}$$

It is a nonlinear version of the bilinear system

$$\left(D_t + D_x^3\right) f \cdot g = 0, \tag{2a}$$

$$D_y\left(D_t + D_x^3\right) f \cdot g = 0, \tag{2b}$$

with the Hirota bilinear operators D_x, D_y and D_t defined by

$$D_x^n D_y^m D_t^k f \cdot g \equiv \partial_{\epsilon_1}^n \partial_{\epsilon_2}^m \partial_{\epsilon_3}^k f(x+\epsilon_1, y+\epsilon_2, t+\epsilon_3) \times g(x-\epsilon_1, y-\epsilon_2, t-\epsilon_3)\big|_{\epsilon_1=\epsilon_2=\epsilon_3=0},$$

which is one of the five systems classified by Hietarinta through his systematic study on the modified Korteweg–de Vries (mKdV) type bilinear equations [24]. The Lax structure, N-soliton solution based on Wronski determinant and bilinear Bäcklund transformation have been established in Ref. [25]. Infinitely many symmetries, the related Kac–Moody–Virasoro type symmetry algebra and symmetry reductions have been presented in Ref. [26].

It is noted that when the field v is zero, Eq. (1) will be reduced to the celebrated mKdV equation

$$u_t + u_{xxx} + 3u^2 u_x = 0, \tag{3}$$

while if u is zero, then we have the potential Boiti–Leon–Manna–Pempinelli (pBLMP) equation

$$v_{yt} + v_{xxxy} + 3(v_x v_y)_x = 0, \tag{4}$$

which is an alternative form of the $(2+1)$-dimensional KdV equation or the asymmetric Nizhnik–Novikov–Vesselov equation. Thus, Eq. (1) is called the coupled mKdV–pBLMP equations.

This Letter is organized as follows. The basic multilinear variable separation approach is applied to the coupled system (1) in Section 2 to derive the variable separation solutions. As a result, the potential quantities u_y and v_y are exactly expressed by the universal formula applicable to all multilinear variable separable systems. Furthermore, the generalized multilinear variable separation method is also applied, and the results show that the potential field u_y (v_y) is rightly the subtraction (addition) of two universal formula with different constant parameters, which is presented in Section 3. In Section 4, interactions between periodic waves are discussed, including the elastic interaction between two semi-periodic waves and non-elastic interaction between two periodic instantons. Different limiting cases are investigated accordingly. The last section is devoted to summary and discussion.

2. Basic variable separation solution

Take an auto-Bäcklund transformation

$$u = (\ln\phi)_x + u_0, \qquad v = (\ln\phi)_x + v_0, \tag{5}$$

where ϕ is an undetermined function of (x, y, t), u_0 and v_0 satisfy the coupled mKdV–pBLMP equations. It is obvious that

$$u_0 = 0, \qquad v_0 = v_0(x,t), \tag{6}$$

where $v_0(x,t)$ is an arbitrary function of the indicated variables, is a particular seed solution of Eq. (1). Substituting Eq. (5) with (6) into Eq. (1) leads to

$$\phi\phi_{xt} + 3\phi\phi_{xx}v_{0x} + 3\phi\phi_x v_{0xx} + \phi\phi_{xxxx} - \phi_x\phi_t - \phi_{xxx}\phi_x - 3\phi_x^2 v_{0x} = 0, \tag{7}$$

and

$$\begin{aligned}&(\phi_{xxxy} + \phi_{xyt} + 3v_{0x}\phi_{xxy})\phi^2 - (\phi_{xxxy} + \phi_{yt})\phi\phi_x \\ &+ (2\phi_x\phi_y - \phi\phi_{xy})\phi_{xxx} \\ &- (\phi_{xt} + \phi\phi_{xxxx})\phi\phi_y + \left(3\phi^2 v_{0xx} - \phi\phi_t - 6\phi v_{0x}\phi_x\right)\phi_{xy} \\ &+ 3\left(2\phi_x^2 - \phi\phi_{xx}\right)v_{0x}\phi_y + (2\phi_t - 3\phi v_{0xx})\phi_x\phi_y = 0,\end{aligned} \tag{8}$$

respectively. It is easy to check that Eq. (8) is identically satisfied under Eq. (7). Therefore, we only need to insert the following variable separation assumption

$$\phi = a_0 + a_1 p + a_2 q + a_3 pq, \tag{9}$$

with constants a_i, $(i = 0, 1, 2, 3)$, and functions $p \equiv p(x,t)$ and $q \equiv q(y,t)$ of the indicated arguments, into Eq. (7), leading to

$$(a_1 + a_3 p)q(\phi\partial_x - \phi_x)(p_t + 3v_{0x}p_x + p_{xxx}) + (a_0a_3 - a_1a_2)p_x q_t = 0. \tag{10}$$

The above equation can be divided into two variable separated equations,

$$p_t + 3v_{0x}p_x + p_{xxx} - (a_0a_3 - a_1a_2)(c_0 + c_1 p) = 0, \tag{11}$$

and

$$q_t + a_1(a_0c_1 - a_1c_0) + \left((a_0a_3 + a_1a_2)c_1 - 2a_1a_3c_0\right)q + a_3(a_2c_1 - a_3c_0)q^2 = 0, \tag{12}$$

where c_0 and c_1 are arbitrary functions of t. An important trick utilized in the MLVSA is that one can take p as arbitrary and solve v_0 from Eq. (11) instead. As a result, we have

$$v_0 = \int \frac{(a_0a_3 - a_1a_2)(c_0 + c_1 p) - p_t - p_{xxx}}{3p_x}\,\mathrm{d}x + C, \tag{13}$$

where C is an arbitrary integration function of t.

Thus, the variable separation solution of Eq. (1) is obtained as

$$u = \frac{(a_1 + a_3 q)p_x}{a_0 + a_1 p + a_2 q + a_3 pq},$$

$$v = \frac{(a_1 + a_3 q)p_x}{a_0 + a_1 p + a_2 q + a_3 pq} + v_0, \tag{14}$$

where v_0 is given by Eq. (13), q is determined by Eq. (12), and p is an arbitrary function of x and t. It is interesting that the potential quantities u_y and v_y read

$$u_y = v_y = \frac{(a_0a_3 - a_1a_2)p_x q_y}{(a_0 + a_1 p + a_2 q + a_3 pq)^2} \equiv U, \tag{15}$$

where U is just the universal quantity valid for a large class of multilinear variable separable systems [16].