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Atomic focusing by quantum fields: Entanglement properties

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ABSTRACT

The coherent manipulation of the atomic matter waves is of great interest both in science and technology. In order to study how an atom optic device alters the coherence of an atomic beam, we consider the quantum lens proposed by Averbukh et al. [1] to show the discrete nature of the electromagnetic field. We extend the analysis of this quantum lens to the study of another essentially quantum property present in the focusing process, i.e., the atom–field entanglement, and show how the initial atomic coherence and purity are affected by the entanglement. The dynamics of this process is obtained in closed form. We calculate the beam quality factor and the trace of the square of the reduced density matrix as a function of the average photon number in order to analyze the coherence and purity of the atomic beam during the focusing process.

1. Introduction

Since the seminal proposal for laser cooling of atoms in dilute gases and atom trapping [2], the manipulation of all atomic motional degrees of freedom based on the atom interaction with external light fields have reached enormous success. Given the recent advances in the manipulation of atoms we now observe a fast evolution of the field both in terms of scientific knowledge and technological applications, like in precision sensors, precise metrology and clocks, lithography, single atom manipulation, trace gas analysis and ultracold chemistry [3]. In addition, the area of quantum information processing has benefited from such advances due to the establishment of precise quantum protocols. From the theoretical viewpoint the modeling of strongly correlated materials and nonequilibrium quantum dynamics are stimulating areas of research.

The dynamics of atomic beams share an intimately close analogy with classical laser light in the paraxial approximation. The Gouy phase discovered and measured in 1890 in the latter context is found in any beam subject to confinement which adds a well-defined phase shift and has implications and applications in many optical systems [4]. The existence of a particle wave analogy to this phenomenon has been first pointed out in Refs. [5] followed by an experimental proposal in Cavity Quantum Electrodynamics (CQED) [6]. Very recently this proposal has stimulated the search for the matter wave Gouy phase in different systems: Bose–Einstein condensates [7], electron vortex beams [8], and astigmatic electron matter waves using in-line holography [9]. The Gouy phase carries intrinsic properties of the initial state and dictates the time scale of the process.

In the present work we explore the quantum version of experimental set up proposed in Ref. [6] in order to show how it may be of use to explore other quantum features as atom–field entanglement, analysis of atomic quantum lenses proposal in [1] to study the discrete nature of the field. The actual measurement of this phenomenon represents a major experimental challenge, since a quantum tomography would be required. We show here however, that the measurement of the covariance matrix of the center of mass atomic wavefunction indicates the presence of entanglement. Purity loss, although far from being an easily measurable quantity is shown to reveal the entanglement dynamics which occurs in the focusing process. We setup a model (within experimental reach) of a focusing and deflection of a nonresonant atomic beam propagating through a spatially inhomogeneous quantized electromagnetic field. The interaction of a nonresonant atom with an electromagnetic field in the so-called dispersive approximation is proportional both to the field intensity and the susceptibility of the atom. Therefore atoms under the influence of such fields may suffer mechanical effects such as deviations in their center of mass motion and deflection. In the present case we will use this property to focus atomic beams. We address the question as to the manifestation of quantum effects in the focusing process. In Ref. [1] the discrete character of the photons was shown to be

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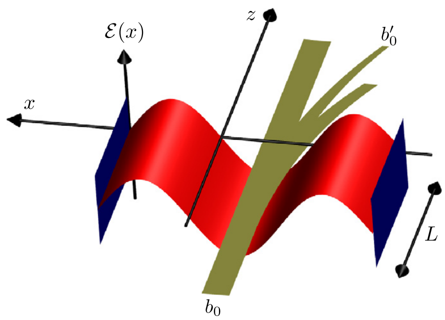

Fig. 1. Quantum lens. A beam of nonresonant atoms propagating initially along the z-axis interacts with the light field in the region $-L \leqslant z \leqslant 0$. Different Fock states deflect the atoms in different directions and focus them at different points. The initial width of the atomic beam is b_0 and b_0' represents its width at the focus.

observable in such experiments. Our aim within a similar scheme is to enlighten another quantum aspect, entanglement. Interaction is the key ingredient to produce entanglement which is an important characteristic of quantum information protocols. We study its behavior in the atomic focusing process.

In Section 2 we present the model which is essentially the same as the one used in Refs. [1,10] with the difference that we calculate the probability amplitude instead of the intensities. Our procedure enable us to determine the density matrix of the system. In Section 3, we present our results, in the covariance matrix and the atom–field entanglement properties as a function of the average photon number $\bar{n}$ showing that one aspect of the classical limit of the field is the suppression of entanglement as $\bar{n}$ increases. This is also apparent in the covariance matrix. The independence of the field's granular nature on the number of photons, shown in Refs. [1,10], occurs because in that model the authors relax the dispersive limit condition. In the present model, we preserve the dispersive limit and the classical limit of the field is a consequence of the disentanglement between atom and field, apparent in the conservation of the initial purity and coherence of the atomic beam.

2. The model

In this section we present a model that permit us focusing an atomic beam and find an expression for the Gouy phase of matter waves that is a connection of this phase with the inverse square of the beam width. We consider an atomic beam propagating through a spatially inhomogeneous quantized electromagnetic field. The atomic beam will suffer deflection and focusing. Different Fock states deflect the atoms in different angles and focus them at different points. We suppose that the atomic beam is initially in a coherent Gaussian state and obtain the equations of motion for the parameters that characterize the structure of the wavepacket. We see that the equations of motion is not consistent if the atomic beam was represented at time by the one Gaussian state without the Gouy phase term.

The model is presented in Fig. 1 in which we use the following [1,10]: consider two-level atoms moving along the Oz direction and that they enter in a region where a stationary electromagnetic field is maintained. The region is the interval $z = -L$ until $z = 0$. The atomic linear momentum in this direction is such that the de Broglie wavelength associated is much smaller than the wavelength of the electromagnetic field. We assume that the atomic center of mass moves classically along direction Oz and the atomic transition of interest is detuned from the mode of the electromagnetic field (dispersive interaction). The Hamiltonian for this model is given by

$$\hat{H}_{AF} = \frac{\hat{p}_x^2}{2m} + g(\hat{x})\hat{a}^\dagger\hat{a}, \tag{1}$$

where m is the atom mass, $\hat{p}_x$ and $\hat{x}$ are the linear momentum and position along the direction Ox, $\hat{a}^\dagger$ and $\hat{a}$ are the creation and destruction operators of a photon of the electromagnetic mode, respectively. The coupling between atom and field is given by the function $g(x) = \alpha\mathcal{E}^2(x)$ where α is the atomic linear susceptibility, $\alpha = \frac{\wp^2}{\hbar\Delta}$, where $\wp^2$ is the square of the dipole moment and Δ is the detuning from nearest atomic resonance. $\mathcal{E}(x)$ corresponds to the electric field amplitude in vacuum. The effective interaction time is $t_L = \frac{L}{v_z}$, where v_z is the longitudinal velocity of the atoms. For simplicity the field distribution in z-direction of length L is assumed to have a rectangular profile as expressed by the Heaviside step functions $\theta(z)$. The initial width of the atomic beam is b_0 and b_0' represents its width at the focus.

The dynamics of the closed system is governed by the Schrödinger equation

$$i\hbar\frac{d}{dt}\big|\Psi(t)\big\rangle = \hat{H}_{AF}\big|\Psi(t)\big\rangle. \tag{2}$$

At $t = 0$ the state of the system is given by a direct product of the state corresponding to the transverse component of the atom and a field state, $|\Psi_{CM}\rangle \otimes |\Psi_F\rangle$. The field state can be expanded in the eigenstates of the number operator $\hat{a}^\dagger\hat{a}$

$$|\Psi_F\rangle = \sum_n w_n|n\rangle, \qquad \sum_n |\omega_n|^2 = 1. \tag{3}$$

When atom and field interact the atomic and field states get entangled. We can then write

$$\big|\Psi(t)\big\rangle = \sum_n w_n \int_{-\infty}^{+\infty} dx\, \psi_n(x,t)|x\rangle \otimes |n\rangle, \tag{4}$$

where

$$i\hbar\frac{\partial}{\partial t}\psi_n(x,t) = \left\{-\frac{\hbar^2}{2m}\nabla^2 + g(x)n\right\}\psi_n(x,t), \tag{5}$$

or, if one defines

$$\big|\Psi_n(t)\big\rangle = \int_{-\infty}^{+\infty} dx\, \psi_n(x,t)|x\rangle, \tag{6}$$

Eq. (5) takes the form

$$i\hbar\frac{d}{dt}\big|\Psi_n(t)\big\rangle = \left[\frac{\hat{p}_x^2}{2m} + g(\hat{x})n\right]\big|\Psi_n(t)\big\rangle. \tag{7}$$

Next, we will use the harmonic approximation for $g(x)$ where we consider that the electric field has a node in the atomic beam axis. In addition, we considered that the width of the transverse atomic beam b_0 is much smaller than the wavelength λ of the field. In this case, as a good approximation, the field creates one square well potential for the atom in the transverse coordinate [1,10]. Therefore we take only the main terms of the Taylor expansion of the function $g(x)$,

$$g(x) \approx g_0 - \frac{g_1^2}{2g_2} + \frac{1}{2}g_2(x - x_f)^2, \tag{8}$$

where $g_0 \equiv g(x=0)$, $g_1 \equiv dg/dx|_{x=0}$, $g_2 \equiv d^2g/dx^2|_{x=0}$, $x_f \equiv -g_1/g_2$ and $\Omega_n^2 = ng_2/m$. The combination of linear and the quadratic contributions of the potential in a binomial reduces the problem to the motion in the harmonic potential $U_n(x) = U_n(x_f) +$