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Generation of macroscopic entangled coherent states with large Josephson junctions

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ABSTRACT

We propose a simple and experimental architecture to generate macroscopic entanglement in a solid system which consists of two large Josephson junctions and a flux qubit. Through quantum measuring of flux qubit, entangled coherent states of two large Josephson junctions are obtained. The concurrence of entangled coherent states can be accommodated by adjusted systematic parameters. We also give a brief discussion on the experimental feasibility of this proposal.

Entangled coherent states have important applications in different areas, which include quantum optic and quantum information [1]. In particular, because of its robustness against single-particle decoherence compared with discrete entangled states, entangled coherent states have attracted much attention and shown powerful in quantum information processing [1]. Therefore, increasing attention has been paid to generating entangled coherent states in various physical systems, which include quantum optic [2], microwave cavity QED [3], trapped ions [4], and Bose–Einstein condensation system [5], but not yet realized experimentally, until now.

In recent years, solid circuit superconducting devices (e.g. Cooper-pair boxes, Josephson junctions, and superconducting quantum interference device (SQUID)) have been proposed as candidates to serve as qubits for a superconducting quantum computer, due to their advantage in design flexibility, large-scale integration, and compatibility to conventional electronics [6]. Moreover, preparing two- or three-particle discrete entangled states have been experimentally implemented with superconducting systems [7]. In addition, generating entangled coherent states with superconducting system have been proposed in Refs. [8,9]. However, owing to the complex structure of these schemes [8] and that the initial state must be in coherent state [9], successful generation of entangled coherent states is still a challenge with currently experimental techniques. In this paper, we will study how to generate entangled coherent states with a simply model, which can be fabricated on a chip down to the micrometer scale. Our proposal only composes two large Josephson junctions (LJJs) and a flux qubit which acts as a coupler. By means of measuring the states of the flux qubit, the entangled coherent states of LJJs are prepared via one-step evolution only. So, comparing with previous theoretical proposals [9], our scheme does not require initial state in coherent state. So, it is easier to implement.

The quantum characters of LJJ have been widely studied [10], in the early years. Using a LJJ coupled to two charge qubits was first proposed in Ref. [11]. Then, generation of entanglement of two charge-phase qubits through LJJ was discussed [12]. Here, we first consider a physical system which uses a flux qubit to couple two LJJs. The Hamiltonian of the LJJ can be written as [13]

$$H_J = E_C N^2 - E_J \cos\gamma, \tag{1}$$

where E_C expresses the charging energy, N is the excess Cooper pairs, E_J denotes the Josephson energy, and γ defines the phase drop across the LJJ. When the LJJ works in the phase regime, we can use a harmonic oscillator model approximately equivalent to LJJ, so the Hamiltonian is

$$H_J = \hbar\omega a^\dagger a, \tag{2}$$

where bosonic operators $a^\dagger = \frac{\xi}{2}\gamma - i\frac{1}{2\xi}N$ and $a = \frac{\xi}{2}\gamma + i\frac{1}{2\xi}N$ with parameter $\xi = (E_J/E_C)^{1/4}$; and plasma frequency $\omega = \sqrt{8E_C E_J}/\hbar$.

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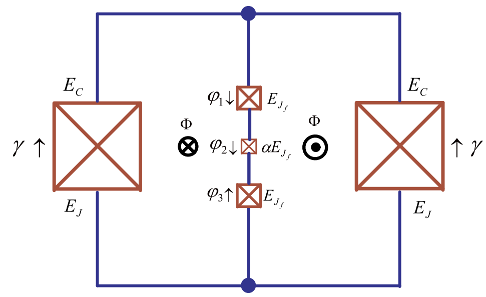

Fig. 1. Schematic diagram of two large Josephson junctions (denoted by a large square with an X inside) coupled by a flux qubit (denoted by three small squares with an X inside). For a three-junction flux qubit, two junctions have the same Josephson coupling energy E_{J_f}, and the third junction has the coupling energy αE_{J_f} ($0.5<\alpha<1$) smaller than that of the other two junctions. All junctions are linked by a superconducting line.

The structure of a flux qubit [14] consists of three Josephson junctions with a superconducting loop, and the charging energy is much smaller than the Josephson coupling energy for each junction. The Hamiltonian of the flux qubit can be described as a two-level system

$$H_f=\frac{1}{2}\varepsilon(\Phi)\sigma_z-\frac{1}{2}\Delta\sigma_x, \tag{3}$$

where $\varepsilon(\Phi)=2I_p(\Phi_0-\Phi)$ is the energy spacing of the two classical current states, I_p is persistent current of the flux qubit, $\Phi_0=h/2e$ is the magnetic-flux quantum, Φ is the external magnetic flux applied to the qubit; Δ is the energy gap between the two states at the degeneracy point; Pauli matrices $\sigma_z=|e\rangle\langle e|-|g\rangle\langle g|$ and $\sigma_x=|e\rangle\langle g|+|g\rangle\langle e|$ are defined in terms of the classical current, where $|g\rangle=|\circlearrowright\rangle$ and $|e\rangle=|\circlearrowleft\rangle$ denote the states with clockwise and counterclockwise currents in the loop.

The model is shown in Fig. 1. Two LJJs of left and right are coupled by a middle flux qubit. According to Kirchhoff's current law, the current of through flux qubit equal to the sum of two LJJ. The fluxoid quantization relation for this circuit is $\gamma+\varphi_1+\varphi_2-\varphi_3+2\pi\Phi/\Phi_0=0$. Due to the Josephson energy of LJJ is larger than other junctions, the Hamiltonian of the total system can be described by

$$\begin{aligned}H&=H_{J1}+H_{J2}+H_f+H_{int}\\&=\sum_{i=1}^{2}\hbar\omega_i a_i^{\dagger}a_i+\frac{1}{2}\varepsilon(\Phi)\sigma_z-\frac{1}{2}\Delta\sigma_x+\sum_{i=1}^{2}g_i'\left(a_i^{\dagger}+a_i\right)\sigma_z,\end{aligned} \tag{4}$$

where the last term describes the spin–boson interaction between the flux qubit and the two LJJs, with the coupling strength $g_i'=I_p\Phi_0/2\pi\xi$. After transformation to the eigenbasis $\{|0\rangle=\cos\frac{\theta}{2}|e\rangle+\sin\frac{\theta}{2}|g\rangle, |1\rangle=-\sin\frac{\theta}{2}|e\rangle+\cos\frac{\theta}{2}|g\rangle\}$ of the flux qubit with the parameter $\theta=2\arctan\frac{\Delta}{\varepsilon(\Phi)}$, the Hamiltonian can be rewritten as

$$H=\sum_{i=1}^{2}\hbar\omega_i a_i^{\dagger}a_i+\frac{\hbar\Omega}{2}S_z+\sum_{i=1}^{2}\hbar g_i'\left(a_i^{\dagger}+a_i\right)(\cos\theta S_z-\sin\theta S_x), \tag{5}$$

where $\hbar\Omega=\sqrt{\varepsilon^2(\Phi)+\Delta^2}$, $S_z=|1\rangle\langle 1|-|0\rangle\langle 0|$, and $S_x=|1\rangle\langle 0|+|0\rangle\langle 1|$. Under the condition $\Omega\gg\{\omega_i,g_i\}$, we can neglect the rapidly oscillating terms, in the interaction picture, the Hamiltonian of the total system can be reduced to

$$H=\sum_{i=1}^{2}\hbar g_i\left(a_i^{\dagger}e^{i\omega_i t}+a_i e^{-i\omega_i t}\right)S_z, \tag{6}$$

where g_i is the effective coupling coefficient given by $g_i=g_i'\varepsilon(\Phi)/\sqrt{\varepsilon^2(\Phi)+\Delta^2}$.

Next, we discuss how to generate macroscopic entangled coherent states with our scheme. Due to the bosonic operators $\{a_i,a_i^{\dagger},I\}$ form a closed Lie algebra, the evolution operator of Hamiltonian (6) can be written in a factorized way [15]

$$U=\prod_i\exp(-if_{i0})\exp(-if_{i1}a_i)\exp\left(-if_{i2}a_i^{\dagger}\right), \tag{7}$$

where f_{i0}, f_{i1}, f_{i2} and f_{i3} are the time-dependent coefficients. By solving equation $i\hbar\partial U/\partial t=HU$ with the initial condition $f_{i0}(0)=f_{i1}(0)=f_{i2}(0)=0$, we get $f_{i0}=g_i^2[(1-e^{-i\omega_i t})/i\omega_i-t]/\omega_i$, $f_{i1}=f_{i2}^{*}=ig_i(e^{-i\omega_i t}-1)S_z/\omega_i$. Suppose the LJJs are initially in their ground states $|0_1\rangle|0_2\rangle$ (the subscripts 1 and 2 represent the left and right LJJ, respectively) and the flux qubit is initially in a superposition state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$. Hence, the state of the total system can be written as $|\Psi(0)\rangle=\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)|0_1\rangle|0_2\rangle$ at $t=0$. Under the unitary operator (7), the initial state of the total system evolves as

$$\left|\Psi(t)\right\rangle=\frac{1}{\sqrt{2}}\left(|0\rangle\left|\alpha_1(t)\right\rangle\left|\alpha_2(t)\right\rangle+|1\rangle\left|-\alpha_1(t)\right\rangle\left|-\alpha_2(t)\right\rangle\right), \tag{8}$$

which is a tripartite entangled state of one flux qubit and two LJJs. Here $|\alpha_i(t)\rangle=e^{\alpha_i(t)a_i^{\dagger}-\alpha_i^{*}(t)a_i}|0_i\rangle$ is a coherent state characterized by the complex variable $\alpha_i(t)=\frac{g_i}{\omega_i}(1-e^{i\omega_i t})$. Then, by means of a unitary transformation, we change the basis states $|0\rangle$ and $|1\rangle$ of the flux qubit back to the original basis states $|g\rangle$ and $|e\rangle$, the quantum state (8) becomes

$$\begin{aligned}&\left|\Psi(t)\right\rangle\\&=\frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2}\left|\alpha_1(t)\right\rangle\left|\alpha_2(t)\right\rangle-\sin\frac{\theta}{2}\left|-\alpha_1(t)\right\rangle\left|-\alpha_2(t)\right\rangle\right)|e\rangle\\&\quad+\frac{1}{\sqrt{2}}\left(\sin\frac{\theta}{2}\left|\alpha_1(t)\right\rangle\left|\alpha_2(t)\right\rangle+\cos\frac{\theta}{2}\left|-\alpha_1(t)\right\rangle\left|-\alpha_2(t)\right\rangle\right)|g\rangle.\end{aligned} \tag{9}$$

When we measure the flux qubit in original basis $\{|g\rangle,|e\rangle\}$, the quantum state of two LJJs collapses into $|\Psi_+\rangle=\frac{1}{\sqrt{2}}(\sin\frac{\theta}{2}|\alpha_1(t)\rangle\times|\alpha_2(t)\rangle+\cos\frac{\theta}{2}|-\alpha_1(t)\rangle|-\alpha_2(t)\rangle)$ or $|\Psi_-\rangle=\frac{1}{\sqrt{2}}(\cos\frac{\theta}{2}|\alpha_1(t)\rangle|\alpha_2(t)\rangle-\sin\frac{\theta}{2}|-\alpha_1(t)\rangle|-\alpha_2(t)\rangle)$ with corresponding to the measured results of the flux qubit in the state $|g\rangle$ or $|e\rangle$. In other word, two kinds of entangled coherent states of LJJ were generated with the outcome possibility of 50%. The measurement of superconducting qubit is widely used to read out the state [16,17]. Here, we can use the method of Ref. [18], which introduces a rf superconducting quantum interference devices coupler to respective mediate the interaction between a flux qubit and the detector, between a flux qubit and a Josephson bifurcation amplifier. This method realizes a ideal quantum measurement of a superconducting flux qubit by a Josephson bifurcation amplifier. Also, another quantum measurement flux qubit method can be used in our scheme. Single-shot readout of a superconducting flux qubit by using a flux-driven Josephson parametric amplifier [19]. And, by continuously monitoring the qubit, quantum jumps between the qubit eigenstates can be observed. All measurements were performed using a dilution refrigerator at the base temperature $T\sim 10$ mK [19].

Obviously, $|\Psi_+\rangle$ and $|\Psi_-\rangle$ are bipartite entangled nonorthogonal states. Here, we use the concept of concurrence for bipartite entangled nonorthogonal states [20]. It is easy to show that the concurrence for $|\Psi_\pm\rangle$ is

$$C_\pm=\frac{|\sin\theta|\sqrt{(1-e^{-4|\alpha_1|^2})(1-e^{-4|\alpha_2|^2})}}{1\pm e^{-2(|\alpha_1|^2+|\alpha_2|^2)}|\sin\theta|}. \tag{10}$$