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Coherent nonlinear quantum model for composite fermions

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ABSTRACT

Originally proposed by Read [1] and Jain [2], the so-called "composite-fermion" is a phenomenological quasi-particle resulting from the attachment of two local flux quanta, seen as nonlocal vortices, to electrons situated on a two-dimensional (2D) surface embedded in a strong orthogonal magnetic field. In this Letter this phenomenon is described as a highly-nonlinear and coherent mean-field quantum process of the soliton type by use of a 2D stationary Schrödinger–Poisson differential model with only two Coulomb-interacting electrons. At filling factor $\nu = \frac{1}{3}$ of the lowest Landau level the solution agrees with both the exact two-electron antisymmetric Schrödinger wavefunction and with Laughlin's Jastrow-type guess for the fractional quantum Hall effect, hence providing this latter with a tentative physical justification deduced from the experimental results and based on first principles.

Perhaps the most spectacular physical concept introduced in the description of the fractional quantum Hall effect (FQHE) is the composite fermion (CF). It consists in an intricate mixture of N_e electrons and vortices in a two-dimensional (2D) electron gas orthogonal to a (strong) magnetic field such that the lowest Landau level (LLL) is only partially occupied. Actually, the CF concept provides an intuitive phenomenological way of looking at electron–electron correlations as a part of sophisticated many-particle quantum effects where charged electrons do avoid each other by correlating their relative motion in the energetically most advantageous fashion conditioned by the magnetic field. Therefore it is picturesquely assumed that each electron lies at the center of a vortex whose trough represents the outward displacement of all fellow electrons and, hence, accounts for actual decrease of their mutual repulsion [1,3]. Or, equivalently, in the simplest case of $N_e = 2$ electrons considered in the present Letter, that two flux quanta $\Phi_0 = hc/e$ are *"attached"* to each electron, turning the pair into a LLL of two CFs with a $6\Phi_0$ resulting flux [2]. The corresponding Aharonov–Bohm quantum phase shift equals 2π. In addition to the π phase shift of core electrons, this is in agreement with the requirements of the Laughlin correlations expressed by the Jastrow polynomial of degree 3 and corresponding to the LLL filling factor $\nu = \frac{1}{3}$ [4,5]. Laughlin's guessed wavefunction for odd polynomial degree was soon regarded as a Bose condensate [6–8] whereas for even degree, it was considered as a mathematical artefact describing a Hall metal that consists of a well-defined Fermi surface at a vanishing magnetic field generated by a Chern–Simons gauge transformation of the state at exactly $\nu = \frac{1}{2}$ [9,10].

Although they provide a simple appealing single-particle illustration of Laughlin correlations, the physical origin of the CF, auxiliary field, and magnetic fluxes remains unclear. In particular, the way they are fixed to particles is not explained. Hence tentative theories avoiding the CF concept like e.g. a recent topological formulation of FQHE [11]. In the present Letter, we show how a strongly nonlinear mean-field quantum model, directly deduced from the experimental results, provides an alternative Hamiltonian physical description, based on first principles, of the debated CF quasiparticle.

Consider the 2D electron pair confined in the x–y plane under the action of the orthogonal magnetic field $\mathbf{B}$. The electrons are situated at $z_{1,2} = x_{1,2} + iy_{1,2}$. We adopt the usual center-of-mass coordinate $\bar{z} = (z_1 + z_2)/2$ and the internal separation $z = (z_1 - z_2)/\sqrt{2}$ and we select odd-m angular momenta $m\hbar$ in order to comply with the antisymmetry of the two-electron orbital wavefunction under electron interchange. The spins are aligned with the magnetic field, and thus in a symmetric state. The eigenstate corresponding to the internal motion is $\Psi_m(z) = u_m(r)e^{im\phi}$ where $z = re^{i\phi}$. We adopt the units of length and energy given by the cyclotron radius $\lambda_c = \sqrt{\hbar/(M\omega_c)}$ and by the Larmor energy $\hbar\omega_L = \frac{1}{2}\hbar\omega_c = \hbar eB/(2Mc)$ where M denotes the effective mass of the electron which may incorporate many-body effects. The eigenstate u_m is given by [5]:

$$\left[\nabla_X^2 + E_m + m - \frac{m^2}{X^2} - \frac{X^2}{4} - \frac{K}{X}\right]u_m = 0. \tag{1}$$

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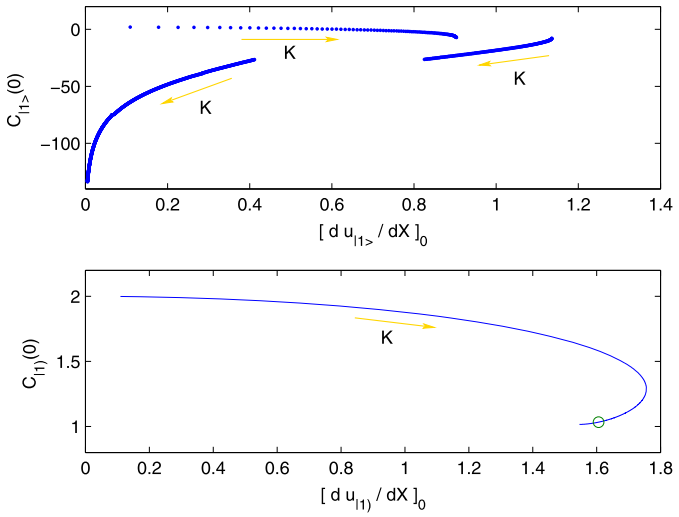

Fig. 1. (Upper panel) The "trajectory" $C_{|1\rangle}(0)$ vs $[du_{|1\rangle}/dX]_0$ for increasing K values within the interval $[0, 15]$ as indicated by the arrows, corresponding to the initial conditions of the LSP problem (5)–(7). (Lower panel) The corresponding "trajectory" defined by the initial conditions of the NSP differential system (5) and (8). The circle indicates the $\nu = \frac{1}{3}$ FQHE solution defined by (4). In both plots, the $K = 0$ free-electron case is defined by the upper left point $du/dX|_0 = 0$ and $C(0) = 2$.

The radial part of the 2D Laplacian operator is $\nabla_X^2 = d^2/dX^2 + X^{-1}(d/dX)$, the energy eigenvalue is E_m and $X = r/\lambda_c$. The dimensionless parameter

$$K = \sqrt{2}\,\frac{e^2/(\epsilon\lambda_c)}{\hbar\omega_c}, \tag{2}$$

where ϵ is the dielectric constant of the semiconductor host, compares the Coulomb interaction between the two particles with the cyclotron energy. Obviously, $K = 0$ corresponds to the free-particle case. Actually, the internal motion could be approximated by the 2D free-particle harmonic oscillator eigenstate $|m\rangle$ as long as $K \leqslant \sqrt{2}$, i.e. $B \leqslant 6$ T in GaAs [5]. However, in the experimental conditions specific to the FQHE the magnetic field is much higher. In Ref. [12] the energy gaps of FQHE states related to samples A and B, at filling factors $p/(2p \pm 1)$, between $\nu = \frac{1}{4}$ and $\nu = \frac{1}{2}$, are shown to increase linearly with the deviation of B from the respective characteristic values $B^A_{\frac{1}{2}} = 9.25$ T, $B^A_{\frac{1}{4}} = 18.50$ T and $B^B_{\frac{1}{2}} = 19$ T (where the superscripts refer to the samples). The corresponding slopes respectively yield the direct measures $M_A = 0.63$, $M_A = 0.93$ and $M_B = 0.92$ of the effective electron mass in units of the electron mass m_e. Indeed, since these masses scale like λ_c^{-1} and hence like $\sqrt{B}$, for they are determined by electron–electron interaction, we have $0.63/\sqrt{9.25} = 0.207 \approx 0.93/\sqrt{18.50} = 0.216 \approx 0.92/\sqrt{19} = 0.211$. Therefore, introducing the parameter κ that accounts for the above experimental results, we have [12]:

$$\frac{M}{m_e} \approx \kappa\sqrt{B}; \quad 0.207 \leqslant \kappa \leqslant 0.216, \tag{3}$$

where B is given in Tesla. Combining now Eqs. (2)–(3) we obtain the FQHE experimental range of the interaction parameter K:

$$11.07 \leqslant K = \frac{4\pi^{3/2}Mc^2}{\epsilon\Phi_0^{3/2}\sqrt{B}} = \kappa\frac{4\pi^{3/2}m_e c^2}{\epsilon\Phi_0^{3/2}} \leqslant 11.56. \tag{4}$$

Eq. (1) is linear and hence dispersive in its free-particle angular-momentum eigenspace. Its stationary solutions are expected to spread out over more and more Slater determinants made up of single particle solutions when the perturbation defined by $K \neq 0$ grows. Our approach is however different. We interpret the mass M as being the effective mass of electrons, without using the CF concept. Our model uses particles of mass M and charge e which interact. In this sense the mass M incorporates the interaction only partially.

Our first step is to consider the $m = 1$, lowest-energy triplet state, and to rewrite Eq. (1) under the form of the following equivalent differential system:

$$\left[\nabla_X^2 + C_{|1\rangle} - \frac{1}{X^2} - \frac{X^2}{4}\right]u_{|1\rangle} = 0, \tag{5}$$

$$\nabla_X^2 C_{|1\rangle} = K\delta(X), \tag{6}$$

where $\delta(X)$ is the Dirac function and the Laplacian ∇_X^2 is 2D in (5), but 3D in (6). The solution of Eq. (6) is

$$C_{|1\rangle}(X) = \mu_{|1\rangle} - KG(X), \tag{7}$$

where the eigenvalue $\mu_{|1\rangle}$ stands for $E_{|1\rangle} + 1$ corresponding to the Larmor rotation at $m = 1$ and $G(X) = 1/X$ is the 3D Green function.

The second step is to replace the 3D with the 2D Laplacian in the Poisson equation (6) such that it becomes electrostatically consistent with the Laplacian used in the Schrödinger equation (1). This means using $G(X) = -\log(X)$ in Eqs. (5) and (7). We will call these equations the linear Schrödinger–Poisson problem (LSP). The LSP is a differential system of second order whose solution $\{u(X), C(X)\}$ is defined by four parameters, conventionally called "initial conditions", by analogy with a dynamical system. We seek a stationary solution defined by $u(0)$, $du/dX|_0$, $C(0)$, and $dC/dX|_0$. (Here, for simplicity, we dropped the subscripts.) Two of these initial conditions, namely, $u(0)$ and $dC/dX|_0$ vanish for any value of K. The first one due to a complete depletion in the vortex trough, and the second one due to the absence of a cusp of the interaction potential at the origin [13]. The phase space associated to the initial conditions is thus reduced to the $C(0)$–$du/dX|_0$ plane. A numerical solution for the LSP (with the 2D Laplacian) is constructed with a shooting method. In Fig. 1(a) we illustrate the "trajectory" in the phase space of the initial conditions for the lowest-energy and most stable free-electron vortex state $|1\rangle$ with increasing K. In practice, to avoid divergences, we consider the initial condition of $X = 4 \times 10^{-4}$ and not at $X = 0$. The discontinuities seen in Fig. 1(a) occur due to the jumps of the ground state solution to higher values of the orbital momentum. The ground state corresponds to $m = 1$ for $K = 0$, but this may change in the presence of the perturbation, the solution eventually spreading over more and more m values when the parameter K increases.

Now, in the third step, we are aiming at reformulating the Poisson equation (6), with a 2D Laplacian, such that the source term becomes the quantum-mechanical charge distribution of one electron as seen by the other electron, represented by the mean-field source term $u^2_{|1)}$,

$$\nabla_X^2 C_{|1)} = u^2_{|1)}. \tag{8}$$

We distinguish the new eigenstates, which are solutions of a nonlinear nature imposed by Eq. (8), by using labels with parentheses instead of kets. We will call Eqs. (5) and (8) the nonlinear Schrödinger–Poisson problem (NSP). The corresponding trajectory of the initial conditions is shown in Fig. 1(b). The spectral coherence of the new solution – i.e. the invariance of its angular momentum with respect to the increase of K – is obvious: instead of discontinuously spreading out in the phase space like in Fig. 1(a) the NSP solution starts spiraling down while keeping its $m = 1$ initial value [13]. No phase transition towards higher angular momenta occurs for $0 \leqslant K \leqslant 15$. E.g. in Fig. 1(b) the trajectory is continuous at $[du_{|1)}/dX]_0 \approx 1.75$ and $C_{|1)}(0) \approx 1.3$. The experimentally relevant values of K, Eq. (4), are included in the small circle.