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Topology optimization of two-dimensional asymmetrical phononic crystals

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ABSTRACT

The multiple elitist genetic algorithm with the adaptive fuzzy fitness granulation (AFFG) is used to design the phononic crystals with large relative bandgap width (BGW) for combined out-of-plane and in-plane wave modes. Without assumption on the symmetry of the unit-cell, we obtain an asymmetrical phononic crystal with the relative BGW which is quite larger than that of the optimized symmetrical structure. With the help of AFFG, the number of the fitness function evaluations is reduced by over 50% and the procedure converges 5 times faster than the conventional evolutionary algorithm to reach the same final fitness values.

1. Introduction

Phononic crystals (PnCs) [1,2] are periodic composite materials that may have full bandgaps, i.e. the frequency ranges within which the propagation of acoustic or elastic waves is totally prohibited. Thus the PnCs can be used to design the structures or devices for damping [3], rectification [4], frequency sensing [5] and waveguiding [6] of sound or elastic waves. In practical applications, it is desirable to design a PnC that has a relative bandgap width (BGW) as big as possible when designing the PnC-based acoustic devices. The topology optimization algorithm provides an effective means for searching the optimal design.

In recent years, the topological approaches including gradient-based or evolutionary methods have been used for many useful designs of photonic crystals (PtCs) [7–12] and PnCs [13–15] for different objective functions and different dimensions. In all these optimization studies except the work by Preble et al. [10] and Gazonas et al. [14], the topology optimization is based on the assumption that the primitive unit-cell has the primary 90° rotational symmetry for cutting down the search space and reducing the optimizing difficulty. However, the symmetry reduction of the system is generally an efficient way to open wider bandgaps in a periodic structure [16–19]. Preble et al. [10] have found a PtC with a large bandgap for the TE mode when the unit-cell lacks high symmetry. Gazonas et al. [14] performed the optimal designs for the acoustic wave using the genetic algorithm (GA). Unfortunately, the shapes of the optimized inclusions are always symmetrical polygons due to the coarse grid of the unit-cell. This limited the generality of the results. Nevertheless, for the PtCs in the single mode [7,10,11] and PnCs in out-of-plane [13,14] or in-plane wave mode [13], the traditional evolutionary algorithm can find the near-optimal solution easily. But it is a challenge for us to perform the topology optimization of the unit-cell for two combined modes without any symmetry. Generally, the following two problems for this optimization need to be solved. Firstly, compared with the optimization of symmetrical unit-cell, non-symmetry of the unit-cell means more possible material distributions within the design domain. If the unit-cell is discretized into $N \times N$ elements, the numbers of the design variables of a symmetrical and an asymmetrical unit-cell are $2^{N\times N/8}$ and $2^{N\times N}$, respectively. Secondly, for an asymmetrical unit-cell, the calculation of the band structure is based on a larger area of the wave vector.

In this paper, without any assumption on the symmetry of the unit-cell, we use a special optimization algorithm in pursuit of the near-optimal solid/solid distribution of a two-dimensional (2D) unit-cell for the combined out-of-plane and in-plane wave modes.

2. Optimization problem and optimization algorithm

The optimization goal here is to maximize the relative BGWs of a complete bandgap appearing among all bands (generally among

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the first several bands) other than the bandgap between two specified neighboring bands. Therefore it can be simply written as

$$\underset{\Sigma}{\text{Maximize:}}\quad d(\Sigma) = \max_{\forall n}\left\{2\cdot\frac{\min_{\mathbf{k}}:\ \omega_{n+1}(\Sigma,\mathbf{k}) - \max_{\mathbf{k}}:\ \omega_n(\Sigma,\mathbf{k})}{\min_{\mathbf{k}}:\ \omega_{n+1}(\Sigma,\mathbf{k}) + \max_{\mathbf{k}}:\ \omega_n(\Sigma,\mathbf{k})}\right\} \tag{1}$$

where Σ denotes the topological distribution within the unit-cell of the PnC; d is the relative complete BGW for combined out-of-plane and in-plane wave modes; and n denotes the serial number of the energy bands (in this paper, we take $n = 1, 2, \ldots, 9$). The calculation of the band structure is based on the finite difference time-domain (FDTD) method [20,21]. In the present work, only the 2D bi-component PnCs with a square lattice are considered. We divide the unit-cell into $N \times N$ pixels, and therefore an $N \times N$ logic matrix is defined. This means that the number of possible structures is $2^{N\times N}$. The value 1 (or 0) of a matrix element means that the corresponding pixel is filled with one (or the other) of the two component materials.

Because of the large number of design variables and non-convexity of the structural optimization problem, the biggest challenge is that much huge computational cost is needed. Besides, how to solve the optimization problem with enough accuracy in the solution space via the evolutionary algorithm becomes pivotal. In the field of structural optimization, in order to reduce the overall computing time, a variety of techniques has been proposed in the literatures by, for instance, using the parallelization techniques [22], improving the algorithm to accelerate convergence [23,24], introducing the fitness approximation techniques [25,26] and using the fuzzy theory [27,28], etc. Perhaps improving GA through different ways is the most popular choice, and parallel computing technique is obviously common in the field of optimization. With these two techniques, a faster process can be obtained and the whole time of the optimization procedure will be significantly reduced. However, another alternative is to estimate some individuals based on the fitness approximation rather than evaluate every individual with the exact fitness function. According to the survey work by Jin and Branke [29], the use of the approximation models in fitness evaluation can reduce the number of fitness calculations most effectively. However, the efficiency of the design optimization relies heavily on the model quality. That is, the lack of effective data and the high dimensionality of the input space will lead to invalid even false fitness approximation of the original exact fitness function.

In such situation, the adaptive fuzzy fitness granulation (AFFG) [28] as a realization of the fuzzy theory in the field of structural optimization is used. It can minimize the number of exact fitness function evaluations (i.e. the calculation of the bandgap of a PnC) by creating a pool of solutions (fuzzy granules) by which an approximate solution may be sufficiently applied to proceed with the evolution. If a given design is sufficiently similar to an existing one, its fitness can be determined by the design in the current pool. In other words, the exact fitness values of many designs in evolution are not needed if they satisfy the relevant condition. Since this approach does not need training data, it cannot fall into the false cases such as false peaks, a large approximation error and large deviation from the true search direction. With this approach, the optimization procedure can produce the solutions with high precision, and the computing time is significantly reduced at the same time [28].

In this paper, we will utilize GA to maximize $d(\Sigma)$. GA has already been used in the topology optimization of PtCs and PnCs. GA (also known as evolutionary algorithms) is a stochastic global search method that mimics the metaphor of natural biological evolution. GA operates on a population of potential solutions (i.e., the unit-cell designs) to produce better and better approximations to the optimal solution. It starts with an initial population according to the objective function and applies the selection, crossover and mutation to produce a new population with higher fitness values. This process is repeated until the algorithm terminates, and then the optimal solution is obtained. In order to preserve more excellent genes from each generation, the multiple elitist genetic algorithm (MEGA) is proposed [30].

Our optimization procedure incorporating MEGA in conjunction with AFFG for the present optimization problem is shown in Fig. 1. The detailed procedure is as follows:

(i) **Initialization.** Start with an initial population with N_p chromosomes, $P_0 = \{X_1^1, X_2^1, \ldots, X_j^1, \ldots, X_{N_p}^1\}$, where $X_j^i = \{x_{j,1}^i, x_{j,2}^i, \ldots, x_{j,r}^i, \ldots, x_{j,m}^i\}$ is the jth individual in the ith generation, with $x_{j,r}^i$ being the rth design variable of X_j^i, and m the number of design variables. Define a multi-set G of fuzzy granules (C_k, σ_k, L_k) according to $G = \{(C_k, \sigma_k, L_k) \mid C_k \in \Re^m, \sigma_k \in \Re, L_k \in \Re, k = 1, 2, \ldots, l\}$ with G being empty in the initial evolution (i.e., the number of fuzzy granules l is zero), where C_k is an m-dimensional vector of centers; σ_k is the width of membership functions of the kth fuzzy granule; and L_k is the life index of the granule. Then, the phenotype of the first chromosome $(X_1^1 = \{x_{1,1}^1, x_{1,2}^1, \ldots, x_{1,r}^1, \ldots, x_{1,m}^1\})$ is taken as the center of the first granule $(C_1 = \{c_{1,1}, c_{1,2}, \ldots, c_{1,r}, \ldots, c_{1,m}\} = X_1^1)$.

Remarks. The distance measurement σ_k that controls the degree of similarity between two individuals is defined as follows

$$\sigma_k = \gamma\frac{1}{(e^{F(C_k)})^\beta} \tag{2}$$

where γ and β are two emphasis parameters which are needed to be chosen carefully. Based on σ_k, the membership function $\mu_{k,r}$ of $x_{j,r}^i$ to each granule member is defined as

$$\mu_{k,r}(x_{j,r}^i) = \exp\left(\frac{-(x_{j,r}^i - c_{k,r})}{(\sigma_k)^2}\right), \quad k = 1, 2, \ldots, l \tag{3}$$

by a Gaussian similarity neighborhood function for each parameter k. If β increases, the distance measurement σ_k and the membership function $\mu_{k,r}$ will decrease, leading to necessity of more exact fitness evaluations. The similar problem exists for γ. The previous study showed that the choice of β and γ dependent on the particular problem being considered [28]. Generally, the values of β and γ are selected between 0 and 0.5 and between 1.0 and 5.0, respectively, in topology optimization of structures [28]. In the present study, the values of these two parameters are selected by numerical tests based on the balance between the acceleration of the optimization procedure and the accuracy of the fitness approximation. Our numerical tests suggest that the choice of $\beta = 0.1$ and $\gamma = 1.0$ can guarantee the average similarity of every new solution to each granule being larger than 90% in the early evolution and 97% in the later evolution with at least 50% of the finite element analysis (FEA) evaluations being reduced. So, in this paper, we select $\beta = 0.1$ and $\gamma = 1.0$.

(ii) **Similarity calculation.** Calculate the average similarity of a new solution $X_j^i = \{x_{j,1}^i, x_{j,2}^i, \ldots, x_{j,r}^i, \ldots, x_{j,m}^i\}$ to each granule G_k of the present granule pool according to $\bar{\mu}_{j,k} = \sum_{r=1}^m \mu_{k,r}(x_{j,r}^i)/m$.