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Waves and energy in random elastic guided media through the stochastic wave finite element method

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ABSTRACT

Energy propagation in random viscoelastic media is considered in this Letter. The forced response of uncertain waveguide subject to time harmonic loading is treated. This energy model is based on a spectral approach called the "Stochastic Wave Finite Element" (SWFE) method which is detailed in this Letter. Assuming that the random properties are spatially homogeneous in the media, the SWFE is a hybridization of the deterministic wave finite element and a parametric probabilistic approach. The proposed model is applicable in a wide frequency band with reduced time consumption. Numerical examples show the effectiveness of the proposed approach to predict the statistics of kinematic and quadratic variables of guided wave propagation. The results are compared to Monte Carlo simulations.

1. Introduction

Guided waves are still a subject of intensive research as such structural forms occur in several engineering areas. A number of numerical methods are under study to deal with their behavior. The Wave Finite Element (WFE) approach and Semi-Analytical Finite Element (SAFE) among others are investigated by many researchers [1–3] in Structural Health Monitoring (SHM) and Non-Destructive Testing (NDT).

The dynamic behavior of waveguides are often investigated using the SAFE technique which considers wave propagation by means of specific shape functions. This method is used for wave propagation and inhomogeneities scattering prediction in some specific structures. The Wave Finite Element (WFE) is a deterministic spectral method dealing with the wave propagation problems of homogenous and periodic structures. This method is considered through the modeling of a subsystem by classical FE, extracting the mass and stiffness matrix by commercial tool and applying the periodicity conditions to establish an eigenvalue problem which describes wave propagation in the system. Mace [4] and Ichchou et al. [5] developed the WFE for structural vibration analysis, Mencik et al. [6] used the wave finite element to describe wave propagation in elastic waveguides. Mead [7] proposed a general theory in order to determine harmonic wave propagation characteristics with second-order eigenvalue problem. Zhong and Williams [8] proposed a structured linearization method using state space eigenvalue problem for large matrices. Houillon et al. [9] used the WFE to study the dynamics of homogenous thin-walled structures with any cross section. Duhamel et al. [10] investigated also the vibration of a uniform waveguide using this technique. Many researchers investigated this method for frequency forced response [11,12] to harmonic excitation. Using the wave-mode representation, this method allows to identify the kinematic variables with small number of degrees of freedom. Renno et al. [13] presented an approach to estimate the forced response of waveguide to general loading using the inverse Fourier transform process. In addition, Mencik et al. [6] applied this method in complex elastic and interconnected structures and offered a general expression of the diffusion matrix. In order to take into account uncertainties in spectral methods, some researches [14,15] tried to predict the statistics of spectral quantities using analytical formulations for longitudinal waves.

A numerical spectral approach describing wave and energy propagation in uncertain media for 1D wave propagation is presented in this Letter. Assuming that the uncertainties are spatially uniform and periodic, we follow the same process than [16] to extend the WFE method to stochastic waveguides. This presented model is a combination between a parametric probabilistic approach and the Wave

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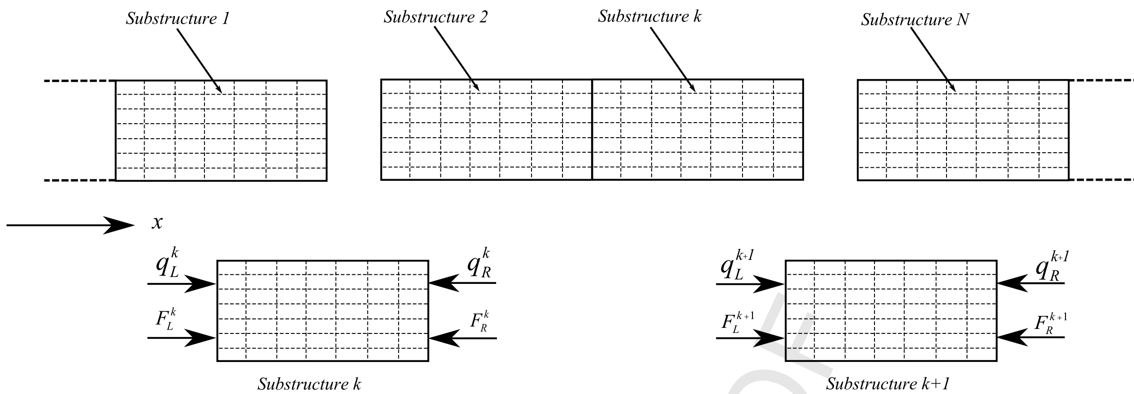

Fig. 1. An illustration of a periodic waveguide.

Finite Element which helps to predict the statistics of kinematic (displacements, forces, ...) and quadratic (kinetic energy density, potential energy density, ...) quantities. The main novelty of this Letter is to investigate the dispersion of kinematic and energy parameters for finite structures through a stochastic wave based method.

The Letter contains five sections. Section 2 is a presentation of the Stochastic Wave Finite Element. A general formulation for forced response and different energy densities is offered in Section 3. In Section 4, the semi-infinite random case is treated. Finally, some numerical experiments dealing with multi-modal waveguide is presented in Section 5. The results are compared to Monte Carlo simulation.

2. Stochastic wave finite element

2.1. Wave finite element

This study concerns a description of the dynamical behavior of a slender structure, as illustrated in Fig. 1, which consists of N identical substructures along a specific direction (x-axis). Each substructure is supposed elastic, linear and dissipative.

Let us consider a finite element model of a given substructure m ($m \in \{1, \ldots, N\}$). The left and right boundaries of the substructure are assumed to contain n degrees of freedom (dofs). The kinematic variables, displacements q and forces F, are noted by (q_L, q_R) and (F_L, F_R), respectively, where subscripts L and R state for *L*eft and *R*ight boundaries.

Let us consider a spectral formulation using the state space variables. This formulation has an advantage in extracting dispersion curves using the symplecticity of transfer matrix. Let's define the kinematic variables in state space as: $u_L = ((q_L)^T(-F_L)^T)^T$ and $u_R = ((q_R)^T(F_R)^T)^T$. The coupling conditions at the boundary of two consecutive substructures m and $m+1$ $(m = 2, \ldots, N)$ are written as: $-F_L^{(m+1)} = F_R^{(m)}$ and $q_L^{(m+1)} = q_R^{(m)}$, so it can be shown that state vectors u_L and u_R are related as:

$$u_L = S u_R \tag{1}$$

where:

$$S = \begin{pmatrix} -D_{LR}^{-1}D_{LL} & -D_{LR}^{-1} \\ D_{RL} - D_{RR}D_{LR}^{-1}D_{LL} & -D_{RR}D_{LR}^{-1} \end{pmatrix}. \tag{2}$$

The stochastic wave finite element formulation dealing with wave propagation in uncertain media is presented. It is a spectral formulation which takes into account uncertainties present in structures. The uncertainties are supposed to be spatially uniform which ensures the periodicity of the structure's variability. The WFE can hence be extended using the first-order perturbation method [16].

In all development of the formulation, each stochastic parameter $\tilde{r}$ will be presented as $\tilde{r} = \bar{r} + \sigma_r\xi$, where $\bar{r}$ is the mean value, σ_r is the standard deviation and ξ is Gaussian centred $(\bar{\xi} = 0)$ and reduced $(\sigma_\xi = 1)$ variable.

Using the state space variables, the kinematic variables through stochastic state vectors $\tilde{u}_L$ and $\tilde{u}_R$ are written: $\tilde{u}_L = ((\tilde{q}_L)^T(-\tilde{F}_L)^T)^T$, $\tilde{u}_R = ((\tilde{q}_R)^T(\tilde{F}_R)^T)^T$ which are defined in the left and the right side of each layer. These state vectors are related by the stochastic transformation matrix $\tilde{S}$ expressed as:

$$(\bar{S} + \sigma_S\xi)[\tilde{u}_L] = [\tilde{u}_R] \iff \begin{pmatrix} \bar{S}_{LL} + \sigma_{S_{LL}}\xi \bar{S}_{LR} + \sigma_{S_{LR}}\xi & \\ \bar{S}_{RL} + \sigma_{S_{RL}}\xi & \bar{S}_{RR} + \sigma_{S_{RR}}\xi \end{pmatrix} \begin{pmatrix} \bar{q}_L + \sigma_{q_L}\xi \\ -\bar{F}_L - \sigma_{F_L}\xi \end{pmatrix} = \begin{pmatrix} \bar{q}_R + \sigma_{F_R}\xi \\ \bar{F}_R + \sigma_{F_R}\xi \end{pmatrix}. \tag{3}$$

The expansion and the projection of Eq. (3) leads to identifying the zero order as:

$$\bar{S} = \begin{pmatrix} -\bar{D}_{LR}^{-1}\bar{D}_{LL} & -\bar{D}_{LR}^{-1} \\ \bar{D}_{RL} - \bar{D}_{RR}\bar{D}_{LR}^{-1}\bar{D}_{LL} & -\bar{D}_{RR}\bar{D}_{LR}^{-1} \end{pmatrix} \tag{4}$$

which is the deterministic symplectic matrix defined in Eq. (2). The first order is also represented as:

$$\sigma_S = \begin{pmatrix} -\bar{D}_{LR}0 \\ -\bar{D}_{RR}1 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{D_{LL}}\sigma_{D_{LR}} \\ \sigma_{D_{RL}}\sigma_{D_{RR}} \end{pmatrix} \begin{pmatrix} 10 \\ -\bar{D}_{LR}^{-1}\bar{D}_{LL} - \bar{D}_{LR}^{-1} \end{pmatrix}. \tag{5}$$

Reference that knowledge that the statistics of the transformation matrix, the stochastic spectral eigenvalue problem can be written as follows: