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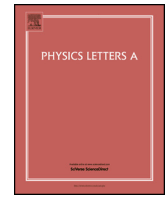

On the quantum behavior of a neutral fermion in a pseudoscalar potential barrier

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ABSTRACT

In this Letter we have studied the quantum behavior of a spin half neutral fermion interacting with a pseudoscalar potential barrier in $(1+1)$-dimensional spacetime. Exact solutions for the corresponding Dirac equation are obtained both for bound and scattering states. The exact energy levels are obtained from the solutions of Dirac equation. The validity of the quasi-classical quantization rule is examined. For the scattering process the transmission and reflection coefficients are exactly calculated. The absence of the Klein's paradox is also discussed.

1. Introduction

It is widely known that relativistic effects can significantly change the quantum mechanical behavior of several physical systems. These changes may be observed in problems associated with bound states or in the scattering processes [1,2]. One can observe the influence of the relativistic effects on the quantum behavior of a physical system in the existence of the bound states, the reduction of the degeneracy of the energy levels, the appearance of the famous Klein's paradox [3–5] and the creation of particles [6]. In addition, when a particle is subjected to an external field, the nature of the coupling plays an important role in its quantum behavior. For example, a charged particle coupled minimally to the electric field via the quadripotential A_μ has a different behavior in comparison with a neutral particle having an anomalous magnetic moment (AMM) and interacting nonminimally with the electric field via the antisymmetric electromagnetic tensor $F_{\mu\nu}$. Certainly, these two modes of coupling influence differently on the quantum dynamics of the particle.

In the present Letter we aim to study the quantum behavior of a neutral fermion in the presence of a pseudoscalar potential barrier by solving the corresponding $(1+1)$-dimensional Dirac equation. This shape of interactions is of interest in relativistic quantum mechanics because it appears when a neutral particle with AMM is subjected to an electric field. In literature we can find a big number of papers addressed to the charged particles coupled minimally to a vector potential and, consequently, the interaction of a charged particle with a vector potential (minimal coupling) is almost understood. In contrast, there are few papers in which the interaction of relativistic particles with the pseudoscalar potential (PSP) is studied. This fact motivates us to consider the $(1+1)$-dimensional Dirac equation with a pseudoscalar potential as an effective model of the usual $(3+1)$ Dirac equation for a neutral particle with AMM in the presence of an electric field. We note that the interaction of a Dirac particle with a PSP has been discussed either by solving the wave equation directly [7–15] or by using the path integral approach [16–19]. It is shown in [15] that bound states solutions can be obtained for the $(3+1)$-dimensional Dirac equation with the pseudoscalar Hulthen potential in contrast to the case of the pseudoscalar Coulomb potential where there is no normalizable wave functions for bound states [13,14]. Bound states solutions are also obtained for the two-dimensional Dirac equation [7–10]. Furthermore, the scattering of a neutral particle by a pseudoscalar step potential is discussed in [11]. In this work we intend to examine the existence of bound states and to study the scattering process in a more general pseudoscalar potential barrier. This generalization allows us to make more conclusions about the Dirac equation with PSP and to understand it's quantum behavior.

The Letter is organized as follows: In Section 2, we show how to obtain the $(1+1)$-dimensional Dirac equation with PSP from the usual $(3+1)$-dimensional equation with a tensor-like potential describing the interaction of a neutral fermion with an electric field. In Section 3,

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we analyze a solvable model where we discuss the existence of bound states and study the scattering process. We discuss, in Section 4, the exactness of the quantization rule obtained from the quasi-classical approximation. The Section 5, is devoted to some particular cases.

2. A physical origin of the $(1+1)$-dimensional Dirac equation

Let us, first, derive the $(1+1)$ Dirac equation with a pseudoscalar potential. To this aim, we consider the motion of a neutral fermion with a nonzero AMM in the presence of an electric field of the form $\vec{E}=\mathcal{E}_0(x)\,\vec{i}$, where $\mathcal{E}_0(x)$ is an arbitrary function of the position component x. This motion is described by the following four-dimensional Dirac equation:

$$\left[i\Gamma^0\frac{\partial}{\partial t}+i\vec{\Gamma}\cdot\vec{\nabla}-m-\frac{1}{2}\mu\sigma^{\mu\nu}F_{\mu\nu}\right]\Psi(t,\vec{x})=0, \tag{1}$$

where Γ^0 and $\vec{\Gamma}\equiv(\Gamma^1,\Gamma^2,\Gamma^3)$ are the usual (4×4) gamma-matrices and

$$\sigma^{\mu\nu}=\frac{i}{2}\left[\Gamma^\mu,\Gamma^\nu\right]. \tag{2}$$

The antisymmetric tensor $F_{\mu\nu}$ is given by

$$F_{\mu\nu}=(\delta_{\mu0}\delta_{\nu1}-\delta_{\mu1}\delta_{\nu0})\mathcal{E}_0(x). \tag{3}$$

Then, if we write the wave function in the form

$$\Psi(t,\vec{x})=\exp\left[-i(Et-k_yy-k_zz)\right]\Gamma^0\Gamma^1\chi(x) \tag{4}$$

we find that $\chi(x)$ is a solution of the equation

$$\left[\Gamma^1E+i\Gamma^0\frac{\partial}{\partial x}-\left(\Gamma^2k_y+\Gamma^3k_z\right)\Gamma^0\Gamma^1-i\mu\mathcal{E}_0(x)-m\Gamma^0\Gamma^1\right]\chi(x)=0, \tag{5}$$

which can be put in the form

$$D_1\chi=D_2\chi, \tag{6}$$

where the operators D_1 and D_2 are given by

$$D_1=i\Gamma^0\frac{\partial}{\partial x}+\Gamma^1E-i\mu\mathcal{E}_0(x)-m\Gamma^0\Gamma^1 \tag{7}$$

and

$$D_2=\left(\Gamma^2k_y+\Gamma^3k_z\right)\Gamma^0\Gamma^1=i\begin{pmatrix}(\sigma_yk_z-\sigma_zk_y) & 0\\ 0 & -(\sigma_yk_z-\sigma_zk_y)\end{pmatrix}, \tag{8}$$

where σ_y and σ_z are the Pauli matrices.

Since $[D_1,D_2]=0$, we can find a common set of eigenfunctions for the operators D_1 and D_2. Furthermore, we can easily show that

$$D_2^2=-\left(k_z^2+k_y^2\right)=-k_\perp^2 \tag{9}$$

and, consequently, the eigenvalues of the operator D_2 are $i\nu k_\perp$, with $\nu=\pm1$. Then, writing $\chi(x)$ in the form

$$\chi(x)=\begin{pmatrix}f(x)\Upsilon_\nu\\ g(x)\sigma_x\Upsilon_\nu\end{pmatrix} \tag{10}$$

where Υ_ν is an eigenvector of $(\sigma_yk_z-\sigma_zk_y)$ with the eigenvalue $\nu k_\perp$, where $\nu=\pm1$, we find that $f(x)$ and $g(x)$ are solutions of the following system of equations:

$$\left(i\frac{\partial}{\partial x}-i\left[\mu\mathcal{E}_0(x)+\nu k_\perp\right]\right)f(x)+(E-m)g(x)=0, \tag{11}$$

$$\left(-i\frac{\partial}{\partial x}-i\left[\mu\mathcal{E}_0(x)+\nu k_\perp\right]\right)g(x)-(E+m)f(x)=0. \tag{12}$$

Taking into account that $\sigma_i\sigma_j=\delta_{ij}+i\epsilon_{ijk}\sigma_k$, we can write this system of equations as follows:

$$\left(\sigma_zE-i\sigma_y\left(-i\frac{\partial}{\partial x}\right)-i\sigma_x\left[\mu\mathcal{E}_0(x)+\nu k_\perp\right]-m\right)\begin{pmatrix}g(x)\\ f(x)\end{pmatrix}=0. \tag{13}$$

Defining the (2×2) gamma-matrices in terms of Pauli matrices

$$\gamma^0=\sigma_z,\qquad \gamma^1=i\sigma_y,\qquad \gamma^5=i\sigma_x, \tag{14}$$