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Trust in the CODA model: Opinion dynamics and the reliability of other agents

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ABSTRACT

A model for the joint evolution of opinions and how much the agents trust each other is presented, using the framework of the Continuous Opinions and Discrete Actions (CODA) model. Instead of a fixed probability that the other agents will decide in the favor of the best choice, each agent considers that other agents might be one of two types: trustworthy or untrustworthy. Each agent its opinion and also the probability for each one of the other agents it interacts with being trustworthy. The dynamics of opinions and the evolution of the trust between the agents are studied. Clear evidences of the existence of two phases, one with strong polarization and the other tending to agreement, are observed. The transition shows signs of being a first-order transition. This happens despite the fact that the trust network evolves much slower than the opinion on the central issue.

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1. Introduction

Opinion dynamics models [1] are usually built on the idea that each agent will influence others, either all of them, if one assumes a complete graph of connections, or a set of all agents that compose its neighborhood. In the most common case, agents somehow change their opinions towards the opinion of those neighbors, either by a simple imitative process in discrete models [2–5] or moving the values of their opinion in the direction of the value of the neighbor opinion, in the continuous models [6,7]. A mixed version exists in the Continuous Opinions and Discrete Action (CODA) model [8,9], where the continuous values are not observed and only a discrete choice is known by the neighbors. CODA model variations have been shown to be equivalent to the continuous models [10] as well as a general case to the discrete models in the literature [11].

However, it is not true that people always tend to copy those they observe. Depending on their own characteristics and those of the person one is interacting with, that observation might have no consequences. Or it could also cause the person who observes to reject the observed characteristic [12]. This idea has been implemented in different ways in different models. No influence has been coded as a threshold in the continuous Bounded Confidence models and as inflexible agents who don't change their opinions in discrete [13–15] and mixed [16] models. The negative influence has also been studied by the introduction of "contrarians" [17–19] who change their opinions as if they wanted to oppose what they observe and other models with contrarian-like effects [20]. In more general terms, the problem of trust in opinion dynamics models has been explored for a number of different models [21–26]. In most cases mentioned here, the trust between agents is not subject to the dynamics of the models, with exception of one case [22]. Distinct approaches to avoid dealing with the trust problem include introducing information following in just one direction [27] or three classes of agents [28], where influence does not happen between the opposed extremes (per example, between leftists and centrists) [29–33].

It is clear, however, that, as our opinion about a certain subject changes, so does our opinion about the person who influenced us. And, in reverse, it is also true that we interpret the same information differently depending on its source [34]. This can lead to people being closer to those who share the same characteristics or opinions, an observed fact known as homophily [35–38]. Here, we will present a generalization of the CODA model where the agents change their opinions about the issue and also about the reliability of the other agents.

This will allow different final states for the society of agents, with both agreement or disagreement being observed in the long run depending on the value of the parameters, as opposed to the original CODA model. In a not strongly connected network, CODA would always lead to disagreement, while, on a complete network, consensus would always happen in the long run. But this is not true on the real world, where many systems end with disagreement even between neighbors, as evidenced by a series of studies of the choices of electors in North-American national and community based elections [39,40].

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2. Trust and likelihood

The original CODA model [10,41] was obtained by assuming that, in a situation where there were two possible choices (or actions), each agent considers there is a fixed probability $\alpha > 0.5$ that each one of its neighbors will have chosen the best alternative. Let the two choices be A and B and let $p_i(t)$ be the probability agent i assigns at time t to the probability that A is the best choice. CODA adopts a fixed likelihood $\alpha \equiv P(OA_j|A)$, representing the chance that, if A is indeed the best choice, when observing agent j, i will observe j chose A, indicated here by OA_j. Assuming that the problem is symmetrical in relation to both choices, that is,

$$P(OA_j|A) = P(OB_j|B),$$

a simple use of Bayes' Theorem will show how $p_i(t)$ is altered. By using log-odd function $\nu \equiv \ln(\frac{p}{1-p})$ (where the agent index and time dependence were omitted for brevity), a simple additive model is obtained. This can be trivially normalized to ν^* so that when A is observed, the agent adds $+1$ to ν^*, and when B is observed, -1 is added. In this case, the choice is defined simply by the sign of ν^*, with positive signs indicating A is chosen. Defined in this way, the model becomes actually independent of α, except for translating the number of steps away from changing opinions back into a probability. For the non-symmetrical case, the likelihoods for choosing A or B could be different and, in this case, the negative and positive steps will no longer have the same size. While, in principle, one could use a normalization where one of the steps is changed to a size of one, there is no strong reason to do that, as the final model will no longer be as simple as the symmetric case.

Contrarians were previously introduced in the symmetrical case simply by assigning a percentage of agents who actually reacted with the opposite sign from the original model [18]. This is actually equivalent to a supposition that the contrarian neighbors are more likely to be wrong than right, that is, they actually have a likelihood of being right given by

$$\mu = 1 - \alpha < 0.5. \tag{1}$$

Of course, the assumption that μ and α add to one is a simplifying one. It actually makes no difference in the dynamics of choices, since, for each agent, in that model, one could define the renormalized number of steps according to the agent own likelihoods. This was due to the fact that each agent treated all other agents it interacted with in the same way, contrarians, in the model, simply believed their neighbors were more likely to be wrong than right.

However, if one wants to introduce the possibility that people trust some people more than others, it makes sense to introduce a trust matrix τ_{ij} that represents how much agent i thinks agent j is likely to be more reliable than untrustworthy. That is, according to agent i there is a probability τ_{ij} that the choices of agent j have a probability $\alpha > 0.5$ of being right (that is that agent j is trustworthy (T)) and a probability $1 - \tau_{ij}$ that those choices have a chance $\mu < 0.5$ of being the best ones (j is untrustworthy U). Each agent believes there are two types of agents, trustworthy agents T who are likely to make the right choices, and untrustworthy U, more likely to choose wrong. In both cases, the agents realize that there is a probability that each of its neighbors will not act according to its character, that is, trustworthy agents have a chance to be wrong $(1 - \alpha)$ and untrustworthy agents have a chance to pick the best choice randomly also (μ).

Obtaining the update rule in this case is a simple application of Bayes' Theorem, as per the framework in the CODA model [9]. Each agent i believes at time t the chance that A is the best choice is given by $p_i(t)$. At that time, it observes an agent j, whom i believes has a probability τ_{ij} of being of the T type, meaning j would chose A with probability α and B with probability $1 - \alpha$. If agent j is a type U, which happens with probability $1 - \tau_{ij}$, it would, instead, choose A with probability μ and B with probability $1 - \mu$.

Assuming that OA_j is observed, that is, that j is observed to prefer A, agent i will update its probability $p_i(t)$ to $p_i(t+1)$ obtained by applying Bayes' Theorem and given by

$$p_i(t+1) = \frac{p[\tau_{ij}\alpha + (1-\tau_{ij})\mu]}{p[\tau_{ij}\alpha + (1-\tau_{ij})\mu] + (1-p)[\tau_{ij}(1-\alpha) + (1-\tau_{ij})(1-\mu)]}, \tag{2}$$

where p is written in the place of $p_i(t)$ for brevity sake. One interesting effect here is that Bayes' theorem also applies to τ_{ij} and the agent i also updates its opinion about how likely j is to be trustworthy. We have a new $\tau_{ij}(t+1)$ given by

$$\tau_{ij}(t+1) = \frac{\tau_{ij}[p\alpha + (1-p)(1-\alpha)]}{\tau_{ij}[p\alpha + (1-p)(1-\alpha)] + (1-\tau_{ij})[p\mu + (1-p)(1-\mu)]}. \tag{3}$$

It makes sense to ask what happens if one adopts the same transformation to log-odds. By calculating $p/(1-p)$, one does get rid of the numerator and taking the logarithm does lead us to an additive model. That is, for $\nu_i(t) = \ln(p/(1-p))$, we have

$$\nu_i(t+1) = \nu_i(t) + \ln\left[\frac{\tau_{ij}\alpha + (1-\tau_{ij})\mu}{\tau_{ij}(1-\alpha) + (1-\tau_{ij})(1-\mu)}\right].$$

If one takes $\tau_{ij} = 1$, that is, certainty that all neighbors are trustworthy, the equation above has a simple constant additive part as second term, the constant size of the step. However, in the general case, the term will change at each interaction since it depends on τ_{ij}. Similarly, the additive term equation for $\theta \equiv \ln(\tau/(1-\tau))$ depends on p and will change with the dynamics. This makes it simpler to work directly with the probabilities p and τ, instead of using log-odds.

In the original CODA model, using p as the main variable was not a good idea also because it approached 0 or 1 too fast, so that computational limitations on the representation of real numbers became a real problematic issue. As a matter of fact, it was usual to obtain values like $1–10^{-300}$, that, in any computer, is indistinguishable from 1.0. The same was not true for ν, meaning it was a much better variable to use. As we will see bellow in the simulation results, when introducing the possibility the neighbor could be untrustworthy, the probabilities do not move to such problematic zones and, therefore, we will not use log-odds from now on, with no trouble other than the lack of simplification of the final model.

Finally, if, instead of OA_j, we have that agent j preference is B, with OB_j being observed, we have, instead of Eqs. (2) and (3), the following update rules:

$$p_i(t+1) = \frac{p[\tau_{ij}(1-\alpha) + (1-\tau_{ij})(1-\mu)]}{p[\tau_{ij}(1-\alpha) + (1-\tau_{ij})(1-\mu)] + (1-p)[\tau_{ij}\alpha + (1-\tau_{ij})\mu]}, \tag{4}$$

and

$$\tau_{ij}(t+1) = \frac{\tau_{ij}[p(1-\alpha) + (1-p)\alpha]}{\tau_{ij}[p(1-\alpha) + (1-p)\alpha] + (1-\tau_{ij})[p(1-\mu) + (1-p)\mu]}. \tag{5}$$

These rules are applied every time another agent is observed, meaning that both the probability and the trust get updated at every iteration. The general effects of one single update can be seen in Figs. 1 and 2. Fig. 1 shows the symmetrical case where $\alpha = 1 - \mu$, showing between the two panels, the difference in effect

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