Physics Letters A 377 (2013) 2378–2383

Contents lists available at SciVerse ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Whistler instability in a semi-relativistic bi-Maxwellian plasma

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ARTICLE INFO

Article history:
Received 18 April 2013
Received in revised form 23 June 2013
Accepted 9 July 2013
Available online 16 July 2013
Communicated by F. Porcelli

Keywords:
Kinetic theory
Whistler wave
Whistler instability
Weibel instability
Temperature anisotropy
Semi-relativistic bi-Maxwellian distribution

ABSTRACT

Employing linearized Vlasov–Maxwell system of equations, the whistler instability is discussed for a semi-relativistic bi-Maxwellian distribution. The dispersion relations are analyzed analytically along with the graphical representation and the estimates of the growth rate and instability threshold condition are also presented in the limiting cases i.e., $\xi_\pm = (\omega \mp \Omega)/k_\parallel v_{t_\parallel} \leqslant 1$ (resonant case) and $\xi_\pm \gg 1$ (non-resonant case). Further for field free case i.e., $B_0 = 0$, the growth rates for Weibel instability in a semi-relativistic bi-Maxwellian plasma are presented for both the limiting cases.

1. Introduction

The presence of relativistic electrons in the magnetosphere induces various types of electromagnetic instabilities due to anisotropy of temperature. This instability arises in a variety of plasmas including fusion plasmas, both magnetic and inertial confinement, as well as in space and astrophysical plasmas. The classical Weibel instability [1] is such an example of an unmagnetized plasma which in the presence of magnetic field generates either whistler or cyclotron maser instability. The Weibel instability, which has been around for several decades is of significant interest since it generates quasi-stationary magnetic fields that can account for seed magnetic fields in laboratory and astrophysical plasmas.

The Weibel instability plays an important role in explaining the generation of cosmic magnetic fields in gamma-ray burst sources and relativistic jet sources, supernovae, and galactic cosmic rays [2–4] as well as the origin of cosmological seed magnetic fields in regions of intense gaseous streaming or temperature anisotropies [5,6]. By using different distributions, the analysis of relativistic Weibel instability has been discussed in detail by several authors [7–13]. A comparative study of Weibel and filamentation instabilities and their cumulative effects has been presented for non-relativistic and weakly relativistic bulk velocities by Bret et al. [14], Lazar et al. [15] and Stockem et al. [16] respectively. Lately, the Weibel instability in quantum plasma has also been studied in linear regime by Haas [17] and in non-linear regime by Haas et al. [18].

Whistler mode emissions were detected inside and outside the Saturn's magnetosphere by plasma wave instruments on Voyager 2 [19,20]. Whistlers are naturally produced in thunderstorms, lightning discharges and also near the north pole which can travel to the south pole along the Earth's magnetic lines of force through the Ionosphere and then return back to the origin. In magnetospheres whistlers are also observed to propagate through self created ducts [21]. In laboratory plasma, whistler mode is used for rf plasma discharge, heating of plasmas in tokamaks [22] and spheromaks [23]. Whistler instability in relativistic regime is a powerful mechanism for producing non-thermal, stimulated radiations (i.e., radio emissions) [24]. The necessary condition for this instability is that the positive gradient along perpendicular velocity should be present in velocity distribution function and such may occur in Solar corona [25], quasi perpendicular shocks [25] and the magnetosheath [26]. The most intense radiations originate from the strongly magnetized auroral regions of the magnetospheres, where the local electron plasma frequency is much less than cyclotron frequency. Such regions are also associated with other planetary magnetospheres and auroras [27,28]. Califano et al. [29] presented the fully relativistic fluid simulation to study the propagation of a relativistic beam in a dense plasma showing that the Weibel instability generates bubble like magnetic structures. The linear and

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0375-9601/$ – see front matter
http://dx.doi.org/10.1016/j.physleta.2013.07.015

non-linear evolution of the electromagnetic beam–plasma instability, in connection to the numerical and experimental results of the laser–plasma interaction, has been discussed in detail by Califano et al. [30] for the non-relativistic and relativistic collisionless inhomogeneous plasma regimes.

Yang et al. [31] calculated the Weibel instability in a relativistic hot magnetized electron–positron plasma and showed that both the decrease in temperature anisotropy and increase in background magnetic field can cause a significant decrease in the growth rate. Shah and Jain [32] studied the excitation of the whistler waves propagating obliquely to the constant magnetic field in a warm and inhomogeneous plasma in the presence of an inhomogeneous beam of suprathermal electrons. The full dispersion relation including electromagnetic effects is derived. In the electrostatic limit the expression for the growth rate is given. It is found that the inhomogeneities in both beam and plasma number densities effect the growth rates of the instabilities. Recently, Lazar et al. [45] discussed the Weibel instability in a magnetized non-relativistic bi-Maxwellian plasma and investigated the threshold conditions for the instability to set in. Mace and Sydora [33] investigated the parallel-propagating whistler instability in a magnetized electron–ion plasma having bi-kappa velocity distributions for a wide range of parameters. Liu et al. [34] and Gary et al. [35] presented linear kinetic dispersion analysis and performed a two-dimensional electromagnetic particle-in-cell simulation to demonstrate a possible excitation mechanism of whistler waves. Schlickeiser et al. [36] discussed the whistler Weibel-like modes in an anisotropic bi-Maxwellian magnetized electron–proton plasma. Palodhi et al. [37] presented a fully non-linear investigation of whistler instability for the non-relativistic case and discussed in detail the transition between the non-resonant Weibel instability and the resonant whistler instability and the formation of non-linear structures.

Recently Zaheer and Murtaza discussed the Weibel instability for the non-Maxwellian distribution functions [38] and for the semi-relativistic Maxwellian distribution function [39] in an unmagnetized plasma. That work was later extended to a magnetized non-relativistic non-Maxwellian plasma [40]. Subsequently Bashir and Murtaza [41], in their review study of plasma waves and instabilities, described the effect of temperature anisotropy on resonant and non-resonant whistler and Weibel instabilities for non-relativistic plasma. In the present Letter, we are investigating the whistler instability in the magnetized anisotropic plasma in the semi-relativistic Maxwellian regimes.

The layout of this Letter is as follows: In Section 2, we use the kinetic theory to calculate the general dispersion relation for a magnetized plasma in both the non-relativistic and the semi-relativistic regimes using anisotropic Maxwellian distributions. We also derive the analytical expressions for the real and the imaginary parts of the dielectric constant for both the momentum distributions in the limiting cases $\xi_\pm \leqslant 1$ and $\xi_\pm \gg 1$. A brief summary of results and discussion is given in Section 3 along with the graphical representation of the whistler instability in a magnetized semi-relativistic bi-Maxwellian plasma for both the limiting cases.

2. Mathematical model

The linear dispersion relation for the transverse electromagnetic electron waves propagating parallel (i.e., $\mathbf{k}(0,0,k_z)$) to the ambient magnetic field B_0, is given by [11,42]

$$\omega^2 - c^2k^2 + \pi\omega_{pe}^2 \int_{-\infty}^{\infty} dp_\parallel \int_0^{\infty} dp_\perp \frac{p_\perp^2}{\gamma(\omega - \frac{k_\parallel p_\parallel}{\gamma m} \mp \Omega)} \times \left\{ \left(\omega - \frac{k_\parallel p_\parallel}{\gamma m}\right)\frac{\partial f_0}{\partial p_\perp} + k_\parallel v_\perp \frac{\partial f_0}{\partial p_\parallel}\right\} = 0 \quad (1)$$

where $\Omega = \Omega_0/\gamma$ is the relativistic cyclotron frequency with $\Omega_0 = eB_0/m_oc$ and $\gamma^2 = 1 + p_\perp^2/m^2c^2 + p_\perp^2/m^2c^2$ and B_0 is along the z-direction. In Eq. (1), the upper and lower signs in the denominator of the integrand correspond to the right-hand and the left-hand circular polarizations, respectively.

In the following, we shall derive the general linear dispersion relations for the non-relativistic and the semi-relativistic Maxwellian momentum distributions [39] i.e.,

$$f_0^N = \frac{1}{2\pi mT_\perp}\frac{1}{\sqrt{2\pi mT_\parallel}}\exp\left[-\frac{p_\perp^2}{2mT_\perp} - \frac{p_\parallel^2}{2mT_\parallel}\right] \quad (2)$$

$$f_0^S = \frac{\exp[\frac{mc^2}{T_\perp}]}{2\pi mT_\perp\sqrt{2\pi mT_\parallel}(1+\frac{T_\perp}{mc^2})} \times \exp\left[-\frac{mc^2}{T_\perp}\sqrt{1+\frac{p_\perp^2}{m^2c^2}} - \frac{p_\parallel^2}{2mT_\parallel}\right] \quad (3)$$

Yoon [9] has studied the Weibel instability with the fully relativistic anisotropic distribution function which in the limit of non-relativistic parallel momentum (i.e., $p_\parallel^2 \ll m^2c^2$ and $p_\parallel^2 \ll p_\perp^2$) gives semi-relativistic distribution function chosen above. We therefore assume that for both the non-relativistic and semi-relativistic cases, the parallel momentum distributions are same having the non-relativistic Maxwellian distribution and the relativistic mass factor only depends upon the perpendicular momentum i.e., $\gamma \approx \gamma_\perp = (1 + p_\perp^2/m^2c^2)^{\frac{1}{2}}$. For $p_\perp^2 \ll m^2c^2$ and $T_\perp \ll mc^2$, the semi-relativistic distribution immediately reduces to the non-relativistic bi-Maxwellian distribution.

Thus performing straight forward $p_\parallel$-integrations with $f_{0\parallel} = \frac{1}{\sqrt{2\pi mT_\parallel}}\exp[-\frac{p_\parallel^2}{2mT_\parallel}]$, Eq. (1) takes the form

$$0 = -\omega^2 + c^2k_\parallel^2 - 2\pi\frac{\omega_{pe}^2}{m^2v_{t\perp}^2}\int_0^\infty \frac{dp_\perp\, p_\perp^3 f_{0\perp}^{N,s}}{\gamma_\perp^2} \times \left\{\frac{\omega}{k_\parallel v_{t_\parallel}}Z(\xi_\pm) - \frac{1}{2}\left(\frac{T_\perp}{T_\parallel} - 1\right)Z'(\xi_\pm)\right\} \quad (4)$$

where

$$f_{0\perp}^N = \frac{1}{2\pi mT_\perp}\exp\left[-\frac{p_\perp^2}{2mT_\perp}\right]$$

$$f_{0\perp}^S = \frac{\exp[\frac{mc^2}{T_\perp}]}{2\pi mT_\perp(1+\frac{T_\perp}{mc^2})}\exp\left[-\frac{mc^2}{T_\perp}\sqrt{1+\frac{p_\perp^2}{m^2c^2}}\right]$$

are the perpendicular distribution functions for the non-relativistic and the semi-relativistic cases respectively and $Z(\xi_\pm)$ is the plasma dispersion function [43] defined as

$$Z(\xi_\pm) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{dS\, e^{-S^2}}{(S-\xi_\pm)}$$

$$\text{with } \xi_\pm = \frac{|\omega \mp \Omega|}{k_\parallel v_{t_\parallel}} \text{ and } v_{t_\parallel} = \sqrt{\frac{2T_\parallel}{m}} \quad (5)$$

$Z'(\xi^\pm)$ indicates the derivative of plasma dispersion function with respect to its argument $\xi_\pm$.

By using the expansion of the plasma dispersion function for the limiting case $\xi_\pm \leqslant 1$

$$Z(\xi_\pm) \simeq i\sqrt{\pi} - 2\xi_\pm + \frac{4}{3}\xi_\pm^3 - \cdots$$

we may write the dispersion relation for R-wave in Eq. (2) as