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Transition on the relationship between fractal dimension and Hurst exponent in the long-range connective sandpile models

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ABSTRACT

The relationships between the Hurst exponent H and the power-law scaling exponent B in a new modification of sandpile models, i.e. the long-range connective sandpile (LRCS) models, exhibit a strong dependence upon the system size L. As L decreases, the LRCS model can demonstrate a transition from the negative to positive correlations between H- and B-values. While the negative and null correlations are associated with the fractional Gaussian noise and generalized Cauchy processes, respectively, the regime with the positive correlation between the Hurst and power-law scaling exponents may suggest an unknown, interesting class of the stochastic processes.

Sandpile dynamics and self-organized criticality (SOC) are known to be exhibited in many natural and social phenomena including earthquakes, forest fires, rainfalls, landscapes, drainage networks, stock prices, traffic jams, and so on. Since the original nearest-neighboring sandpile model was introduced by Bak et al. [1,2], various numerical and analytical studies of modified sandpile models have been a considerable subject of researches, e.g. [3–11]. Among them, the *annealed* random-neighbor sandpile models where an avalanche can propagate within the system were first (perhaps) proposed by Christensen and Olami [4] and then extensively studied on a long-range connected (small-world) network by, for example, de Arcangelis and Herrmann [7], Lahtinen et al. [9], and Chen et al. [10,11].

We have previously proposed a *long-range connective sandpile* (LRCS) model by introducing randomly remote connections between two separated cells [10–13]. For a square lattice of L by L cells, we randomly throw sands, one at a time, onto the grid. In the original Bak–Tang–Wiesenfeld (BTW) sandpile model, once the total amount of the accumulated sands on a single cell reaches the threshold amount of four, they will be redistributed to the four adjacent cells (the nearest neighbors) or lost off the edge of the grid. Our modified LRCS model differs from the BTW model in view of releasing toppled grains to four nearest neighboring cells. The modified rule of randomly internal connections is very similar to the implementation of Watts and Strogatz [14]. For any particular cell, when the accumulated grains exceed the threshold and redistribution occurs, one of the original nearest neighbor connections confronts a chance with the *long-range connective probability* P_c of redirecting to a randomly chosen, distant cell and so the original connection is replaced by a randomly chosen mesh that may be far from the toppling cell. For a scheme of the distribution process of the LRCS model please refer to Fig. 1. We have furthermore assumed that P_c depends strongly on topographic change induced by the last event [11–13]. Consider that topographic height of the sandpile $\mathbf{x}$ at the iteration step t is $\boldsymbol{Z}_t(\mathbf{x})$. At the next iteration step $t+1$, due to the throw of single grain on the grid, it changes from $\boldsymbol{Z}_t(\mathbf{x})$ to $\boldsymbol{Z}_{t+1}(\mathbf{x})$. Therefore, total change in the topographic height of the sandpile is $\Delta Z(t+1) = \sum_{x_l} |Z_{t+1}(x_l) - Z_t(x_l)|$. We then define $P_c(t+1) = [\Delta Z(t+1)/\alpha L^2]^3$. The meaning for the coefficient α is basically like the normalization constant, which makes the value of the connective probability P_c range between 0 and 1. The simulation throughout this study was performed in the "stop-and-go" mode. The LRCS model after a large avalanche can thus evoke a high value of connective probability P_c, motivated by that a more active earthquake fault system will have higher probability to establish long-range connection due to the fault activity, the change in pore fluid pressure or the dynamic triggering of seismic waves. By using such self-adapted probability threshold P_c of remote connection, the self-adapted LRCS model demonstrates a state of *intermittent criticality* [15–17], in which the

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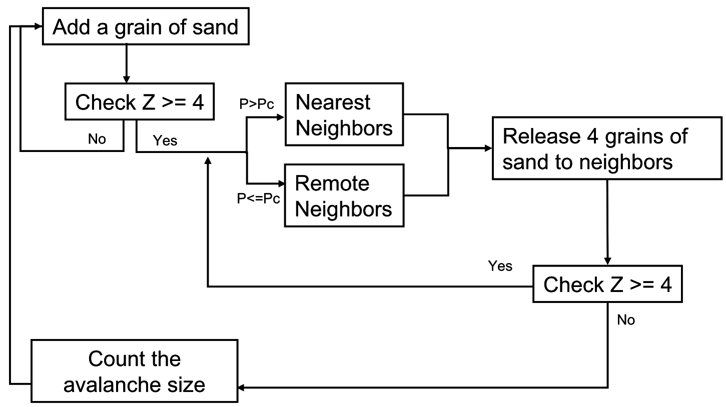

Fig. 1. Flowchart of the LRCS model illustrating the criterion about random distribution to remote site of toppling sand.

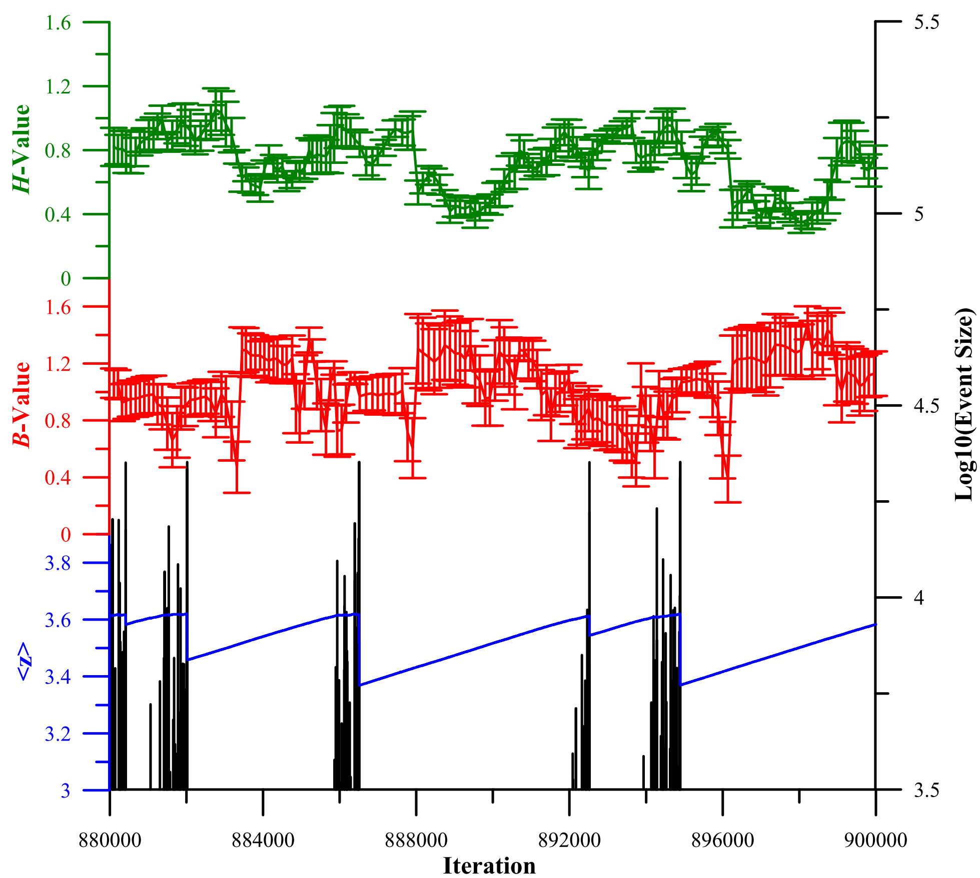

Fig. 2. Simulation for a square lattice of 150 by 150 cells. Blue line represents the dynamic variable $\langle Z\rangle(t)$ of the average topographic height of the LRCS model. Green and red lines are the Hurst exponent H of avalanche sizes and the power-law exponent B of frequency-size distribution, respectively. Error bars show the 95% confidence intervals. Also shown are the time occurrences of avalanches with sizes > 3162 (black bars). (For interpretation of colors in this figure, the reader is referred to the web version of this Letter.)

sandpile quasi-periodically approaches and retreats from the critical state.

In the LRCS model with self-adapted P_c, the dynamic variable of the spatially averaged amount of grains on board $\langle Z\rangle(t)$ $(= (\sum_{i=1}^{L^2} Z_i(t))/L^2$, blue lines in Figs. 2 and 3) is often punctuated towards smaller values by large events (black bars in Figs. 2 and 3). The large fluctuation in $\langle Z\rangle(t)$ is an important feature mimicking the intermittent criticality [15–20]. Large avalanches are then followed by a period of quiescence and a new approach back toward the critical state (Fig. 2) [11–13]. Such process is similar to the dynamical process of the earthquake fault system which repeats by reloading energy and rebuilding correlation lengths towards criticality and the next great event [18–20,33,34]. For more details about the LRCS model, we refer the readers to our previous papers [11–13].

In this Letter we investigate the temporal variations in the power-law exponent B of the frequency-size distributions and in the Hurst exponent H of avalanche sizes for various system sizes L of the LRCS models. To trace variations in B and H with respect to time the sliding window technique was used. We selected 500 events for every time window to calculate Bs and Hs, and then shifted 50 events to calculate the successive B- and H-values of the next window. For the calculation of B, we applied the data binning technique proposed by Christensen and Moloney [21] to