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Thermodynamics of a two-dimensional interacting Bose gas trapped in a quartic potential

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ABSTRACT

We have studied the Bose–Einstein condensation (BEC) of an interacting Bose gas confined in a two-dimensional (2D) quartic potential by using a mean-field, semiclassical two-fluid model. A thermodynamic analysis including the chemical potential, condensate fraction, total energy, and specific heat has been carried out by considering different values of the interaction strength. Finally, we have found that the behaviour of the condensate fraction and specific heat of quartically trapped bosons differs from those of bosons trapped in a harmonic potential.

1. Introduction

The physics of confined, Bose–Einstein condensed systems has attracted much attention since the experimental realization of BEC in dilute atomic gases [1–3]. In these systems, the presence and character of the condensate are strongly influenced by the form of the confinement and the dimensionality of space.

In most of the theoretical studies on cold gases of atoms, the trapping potential is harmonic due to its relevance to the experiments. However, the experimental realization of a trap with different functional form than quadratic [4,5] has increased the interest in anharmonic trapping potentials. These anharmonic traps can be either of the quartic or harmonic plus quartic form and introduce many novel phases which cannot be observed in a pure harmonic trap [6]. For example, introducing an additional quartic anharmonicity to the harmonic trap can lead to a vortex lattice with a central hole or a giant vortex in a rapidly rotating Bose–Einstein condensate [7]. Furthermore, as Gygi et al. [8] have pointed out, quartic traps provide a better prerequisite for the experimental observation of quantum phase transitions of ultracold bosonic atoms in optical lattices. Later, the same trap geometry has been considered in the studies of non-interacting systems of lattice bosons [9] and free bosons [10]. Most recently, a comprehensive thermodynamic analysis has been carried out for the BEC of an ideal Bose system trapped in D-dimensional quartic potentials [11]. In another study, Chaudhary et al. [12] have studied the static and dynamic properties of a Bose–Einstein condensate in a quartic trap for one, two, and three dimensions.

Most of the studies cited above focused on rotating and lattice systems [4–9]. The others which dealt with the bosons trapped in quartic potentials examined either a non-interacting condensate [10,11] or a weakly interacting condensate at zero temperature [12]. Unlike all these studies, we are interested in a non-rotating condensate trapped in a 2D quartic potential and, for such a system, we study the BEC of weakly interacting bosons at finite temperature.

Since the spatial degrees of freedom affect the properties of phase transitions and the nature of collective excitations, the reduced dimensionality alters the physics of ultracold trapped atomic gases [13]. A well-known consequence of this is that, unlike in three dimensions, a homogeneous 2D ideal Bose gas does not undergo BEC at finite temperatures in the thermodynamic limit [14,15]. However, the presence of a trapping potential stabilizes the condensate against long-wavelength fluctuations and allows the possibility of BEC below a certain transition temperature [16]. It is also worth pointing out the effect of two-body interactions on the nature of the phase transition in reduced dimensionality. It is well accepted that there is no BEC for a 2D uniform system of interacting bosons at any finite temperature [15] although a superfluid phase transition in the form of a Berezinskii–Kosterlitz–Thouless (BKT) transition [17,18] can be shown to take place.

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This superfluid phase is characterized by the presence of a quasi-condensate [19–21].

Recent advances in producing 2D atomic gases trapped in optical potentials [13,22–25] have stimulated considerable interest in the low temperature phase diagrams of these systems. In particular, the question of whether a true condensation occurs in a trapped, weakly interacting, 2D Bose gas has been strongly debated [26,27]. In the phase diagram of such a system, a true BEC phase is expected at ultralow temperature for which the system is phase coherent over its full extension. With the increase of temperature one meets the quasi-condensate, superfluid regime, where phase fluctuations due to phonons dominate. Superfluidity can exist with algebraically decaying order and vortices are found only in the form of bound pairs analogous to the uniform case. At higher temperatures, these vortex pairs break and consequently vortices become unbound and free above the BKT transition temperature. Recently experimental evidence for the observation of the BKT transition in a cloud of ^{87}Rb atoms was reported in harmonically trapped quasi-2D systems [24,28–30].

Motivated by recent interest in anharmonic traps and also by novel properties of 2D Bose gases, we study the BEC of an interacting gas confined in a 2D quartic trap. Trapping potentials of this form can be achieved experimentally by superimposing the counter propagating laser beams [10]. Quartic potentials are typically considered as a method for stabilizing a Bose–Einstein condensate at fast rotation. On the other hand, it is seen that these potentials also lead to different and interesting results in the properties of the non-rotating condensates. It is found that the stability of the condensate increases when switching from a harmonic to a harmonic plus quartic potential [31]. It is also concluded that a quartic contribution to the harmonic potential can prevent the depletion of the condensate despite the repulsive interactions [32]. In our study, we find that this conclusion is also true in the case of a pure quartic trap. All these studies reveal that quartic potential is an appropriate form of trapping to obtain more stable condensates in interacting Bose systems.

In the present work, we employ a mean-field, semiclassical two-fluid model [33] to discuss the finite temperature properties of weakly interacting bosons trapped in a 2D quartic potential. The two-fluid model provides a successful framework for describing the effects of two-body interactions on the temperature dependent thermodynamic properties of a Bose–Einstein condensed system. The satisfactory results of the model have been reported by several authors for the harmonic traps in two [34] and three [35,36] dimensions, and for the power-law traps in one dimension [37].

In our study, we find that as the strength of interactions is increased, the temperature dependent behaviour of the system deviates from the non-interacting case noticeably. Moreover, it is also seen that the temperature dependence of the condensate fraction and specific heat vary significantly depending on whether the form of the confining potential is harmonic or quartic. On the other hand, we did not analyze the BKT transition which requires an appropriate theoretical description beyond the two-fluid model for a more systematic investigation in a wider range of temperature and values of the interaction strength. But we believe that the BKT transition in anharmonic traps will be studied both experimentally and theoretically in the near future. Therefore, it is important to investigate the finite temperature properties of the Bose gas in anharmonic traps whose results can constitute a basis for these future studies.

The Letter is organized as follows. In Section 2, we outline the two-fluid model and approximations to carry out a thermodynamic analysis. Section 3 is devoted to the discussion of our results. Finally, the summary and concluding remarks are given in Section 4.

2. Two-fluid semiclassical model for bosons in a quartic trap

The general form of the quartic oscillator potential, which is used as a confining potential in the present work, is as follows [38]

$$V_{ext}(\vec{r}) = \lambda(\vec{r}\cdot\vec{r})^2, \tag{1}$$

where λ is an experimental parameter related to the laser beam characteristics [10]. Harmonic traps with such an additional quartic anharmonicity were realized experimentally in 2004 [4,5]. The transition temperature T_0 for the non-interacting 2D Bose gas in such a quartic trap was calculated in Ref. [10] by using the semiclassical density of states,

$$k_B T_0 = \left[\frac{Nh^2\sqrt{\lambda}}{2\pi^2 m\Gamma(3/2)\zeta(3/2)}\right]^{2/3}, \tag{2}$$

where m is the mass of bosons, $\Gamma(x)$ and $\zeta(x)$ are the gamma and Riemann zeta functions, respectively. In a similar vein, the condensate fraction N_0/N and the total energy of the non-interacting system for $T < T_0$ ($\mu = 0$) are found to be

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_0}\right)^{3/2}, \tag{3}$$

and

$$\frac{E}{Nk_B T_0} = \frac{3}{2}\frac{\zeta(5/2)}{\zeta(3/2)}\left(\frac{T}{T_0}\right)^{5/2}. \tag{4}$$

We note that Eqs. (2) and (3) are consistent with the results of Ref. [16] where the transition temperature and condensate fraction are calculated for an ideal Bose gas confined by a 2D power-law trap.

Hereafter, we shall discuss how the above picture is modified when the short-range interaction effects are introduced into the system. The ground state properties of trapped weakly interacting Bose gas at zero temperature are described by the Gross–Pitaevskii (GP) equation [39]. At finite temperatures, it becomes necessary to take into account the interactions between the non-condensed and condensed particles. In this case, the condensate wave function $\Psi(r)$ satisfies the GP equation in the presence of non-condensed particles [33],

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r}) + V_{ext}(\vec{r})\Psi(\vec{r}) + 2gn_1(\vec{r})\Psi(\vec{r}) + g\Psi^3(\vec{r}) = \mu\Psi(\vec{r}), \tag{5}$$

where μ is the chemical potential, g is the repulsive, short-range interaction strength, and $n_1(r)$ represents the average non-condensed particle distribution. The interaction strength g can be treated as a parameter, as in Ref. [34]. As we consider a 2D system in the present study, we shall be interested in the cylindrically symmetric solution with $V_{ext}(\vec{r}) = \lambda r^4$ and $\Psi(\vec{r}) = \Psi(r)$.

Since the condensate is described by means of the GP equation in the two-fluid model, as seen in Eq. (5), the short-range interparticle interactions considered here are actually weak interactions. On the other hand, the corresponding model treats the non-condensed particles as non-interacting bosons in an effective potential [40]

$$V_{eff}(r) = V_{ext}(r) + 2gn_1(r) + 2g\Psi^2(r), \tag{6}$$

and the density of the non-condensed particles is given semiclassically by [34]

$$n_1(r) = \frac{1}{(2\pi\hbar)^2}\int \frac{d^2p}{\exp\{\beta[\frac{p^2}{2m} + V_{eff}(r) - \mu]\} - 1}, \tag{7}$$