Physics Letters A 375 (2011) 463–467

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Chaotification for a class of cellular neural networks with distributed delays ☆

Na Liu, Zhi-Hong Guan *

Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, 430074, PR China

ARTICLE INFO

Article history:
Received 4 May 2010
Accepted 20 July 2010
Available online 23 September 2010
Communicated by A.R. Bishop

Keywords:
Chaotification
Cellular neural networks
Distributed delays
Lyapunov exponent

ABSTRACT

In this Letter, the chaotification for a class of cellular neural networks with distributed delays is studied. On the basis of the largest Lyapunov exponent, the sensitivity to the initial conditions is studied for the distributed delays with kernel being weak and strong. Some theoretical results about the chaotification for the neural network with distributed time delays are derived. Finally, two numerical simulations are presented to illustrate the effectiveness of the theoretical results.

1. Introduction

Chaos, when under appropriately monitoring, can provide a system designer with a variety of special properties, richness of flexibility, and a cornucopia of opportunities. This provides a strong motivation forth a current research on chaotification of dynamical systems. As a result, many research works have focused on the generation of complex chaotic systems. Feedback control design procedure was suggested in [1], which can rearrange all the Lyapunov exponents of the controlled system according to the user's desire, for any given n-dimensional discrete-time smooth nonlinear dynamical system that could be originally nonchaotic or even asymptotically stable. G. Chen summarized the methods of generating chaos simply either in discrete systems or continuous systems [2]. Furthermore, switching piecewise-linear controllers were proposed to create chaotic attractors [3–5], and a lot of letters devoted to the creation of multi-scroll chaotic attractors [6–8]. In [9], X. Wang and G. Chen proposed a systematic design approach based on time-delay feedback for chaotification of a system. Most recently, Guan and Liu [27] investigated the generating chaos for discrete time-delayed systems via impulsive control.

Time-delay arguments arise in many realistic models of problems in science, engineering, and medicine, where there is a time lag or after-effect. Time-delay is always divided into two parts: one is discrete time-delay, and the other is distributed time-delay. Time-delay can make abundant complex dynamical behaviors, which couldn't be generated by ordinary differential functions. Most authors have focused their attentions on the study of the stability, period solutions, and other complex dynamical behaviors of the time-delays systems, and have obtained a lot of achievements [10–17]. Some letters were devoted to systems with discrete delays and few was devoted to systems with distributed delays.

In the last decades, neural networks have attracted a lot of interests, because they can solve many problems that people cannot solve using custom methods. Many authors have paid much attention to the research on the theory and application of the cellular neural networks. The stability of cellular neural networks with discrete time delays has been discussed in [18–21]. The authors in [22–26] have considered the stability of the cellular neural networks with distributed delays. Up to now, to the best of our knowledge, few authors study the chaotic behaviors for the cellular neural networks with distributed delays. In this Letter, we will analyze a class of cellular neural networks with distributed delay, and obtain some theoretical results about the emergence of chaos for the delayed networks.

This Letter is organized as follows. Model description of the neural network and some primary knowledge about the kernel are introduced in Section 2. Section 3 discusses the chaotification for a class of cellular neural networks with distributed delays in details. Some theoretical results are derived and two examples are presented to illustrate the effectiveness of the theoretical results respectively, when the kernel is the weak one or the strong one. Finally, concluding remarks are drawn in Section 4.

2. Model description and preliminaries

In this Letter, a class of cellular neural networks model with distributed delays is considered as following

☆ This work was supported by the National Natural Science Foundation of China under Grants 60873021, 60973012, 60973039, 61073026 and the International Science and Technology Cooperative Project under Grant 2008DFA12150.

* Corresponding author.
E-mail address: zhguan@mail.hust.edu.cn (Z.-H. Guan).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2010.07.039

$$\dot{x}_i = -a_i x + \sum_{j=1}^{n} w_{ij} f_j\big((g_j * x_j)(t)\big) + p_i, \quad i = 1, \ldots, n \tag{1}$$

and the system (1) can be rewritten in vector-matrix notation of the form

$$\dot{x} = -Ax + Wf\big((g * x)(t)\big) + P, \tag{2}$$

where $x = (x_1, x_2, \ldots, x_n)^\top \in R^n$, $A = \mathrm{diag}(a_1, a_2, \ldots, a_n) > 0$, $W = (w_{ij})_{n\times n}$, and $P = (p_1, p_2, \ldots, p_n)^T \in R^n$ are constant matrices. $f = (f_1, f_2, \ldots, f_n)^T$ and it is assumed that, for $j = 1, \ldots, n$,

(H1) $f_j : R \to R$ are continuous and differential nonlinear functions;
(H2) f_j are bounded in R.

The convolution $g * x$ is defined

$$(g * x)(t) = \int_0^\infty g(s)x(t-s)\,ds,$$

and the kernel $g : [0, \infty) \to [0, \infty)$ satisfies the following normalization assumption:

$$\begin{cases} \int_0^\infty g(t)\,dt = 1, \\ \int_0^\infty sg(s)\,ds < \infty. \end{cases}$$

The commonly used kernel $g(t)$ is always defined Gamma-function, i.e.,

$$g(t) = \frac{\alpha^m t^{m-1} e^{-\alpha t}}{(m-1)!}, \quad m = 1, 2, \ldots, \ \alpha > 0.$$

Subsequently, a parameter $\tau > 0$ is introduced to measure the time delay, which is defined as

$$\tau \triangleq \int_0^\infty t g(t)\,dt,$$

and it can be calculated that

$$\tau = \int_0^\infty t \frac{\alpha^m t^{m-1} e^{-\alpha t}}{(m-1)!}\,dt = \int_0^\infty \frac{\alpha^m t^m e^{-\alpha t}}{(m-1)!}\,dt = \frac{m}{\alpha}, \quad m = 1, 2, \ldots.$$

Especially, when $m = 1$, $g(t)$ is called the weak generic kernel and

$$g(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}}.$$

When $m = 2$, $g(t)$ is called the strong generic kernel and

$$g(t) = \frac{t}{\tau^2} e^{-\frac{t}{\tau}},$$

which has a particular time in the past, namely τ times units ago, is more important than any other since this kernel achieves its unique maximum when $t = \tau$. This kernel can be viewed as a smoothed out version of the case $g(t) = \delta(t - \tau)$, i.e.,

$$(g * x)(t) = \int_0^\infty g(s)x(t-s)\,ds = \int_0^\infty \delta(s-\tau)x(t-s)\,ds = x(t-\tau),$$

which gives rise to the discrete delay model. The weak generic kernel and the strong generic kernel are shown in Fig. 1.

Besides, it is assumed that all solutions of system (1) satisfy the following initial conditions:

$$x_i(t) = \varphi_i(t), \quad t \in (-\infty, 0],\ i = 1, 2, \ldots, n. \tag{3}$$

It is well known that by the fundamental theory of functional differential equations, system (1) has a unique solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^\top$ satisfying the initial condition (3).

Definition 1. System (1) is said to be uniformly bounded, if for any constant $\gamma > 0$ there exists a constant $\beta = \beta(\gamma) > 0$ such that for any $t_0 \in R_+$ and $\phi \in C(-\infty, 0]$ with $\|\phi\| < \gamma$, one has

$$\big|x_i(t, t_0, \phi)\big| \leqslant \beta, \quad t \geqslant t_0,\ i = 1, 2, \ldots, n.$$

3. Chaotification of neural networks with distributed delays

It's clear that if a system is sensitive to the initial conditions, then the system is chaotic, provided that the corresponding system is bounded. The sensitivity is always measured by the largest Lyapunov exponent at the initial conditions, therefore, the largest Lyapunov exponent at the initial point must be calculated. If the parameters of a system are valued, then the Jacobian matrix at the initial point can be figured out. Hence, when the Jacobian matrix is a parameter matrix, the Lyapunov exponents are equal to the eigenvalues of the Jacobian matrix. If the largest Lyapunov exponent is positive, then the system is sensitive to the initial conditions.

Firstly, we discuss the boundedness of the cellular neuron network (1).

Lemma 1. *For the cellular neuron network with distributed delays (1), suppose that the output of the cell f_j ($j = 1, 2, \ldots, n$) satisfy the hypotheses* (H1) *and* (H2)*, then all solutions of system (1) remain bounded for $t \in [0, \infty)$.*

Proof. It is easy to observe that all solutions of system (1) satisfy differential inequalities of the form

$$-a_i x(t) - \xi_i \leqslant \dot{x}_i(t) \leqslant -a_i x(t) + \xi_i, \tag{4}$$

where

$$\xi_i = \sum_{j=1}^{n} |w_{ij}| \sup_{s\in R} \big|f_j(s)\big| + |p_i|.$$

From (4), one can easily prove that solutions of the system (1) remain bounded for $t \in [0, \infty)$. This completes the proof of Lemma 1. $\square$

After having obtained the boundedness of the cellular neural network (1), subsequently, we study its sensitivity to the initial conditions. Since the weak kernel and strong kernel are frequently seen in the literature on delay differential equations, in this section, we will seek the chaotification of the cellular neural networks with distributed delays and two situations are considered: when the g is the weak kernel and when the g is the strong kernel.

3.1. When g is the weak kernel

When g is the weak kernel, for system (2), let

$$y = (g * x)(t) = \int_0^\infty g(s)x(t-s)\,ds = \int_{-\infty}^{t} \frac{1}{\tau} e^{-\frac{t-s}{\tau}} x(s)\,ds,$$

then

$$\dot{y} = \frac{1}{\tau}\big(x(t) - y(t)\big). \tag{5}$$

From (2) and (5), we can easily obtain the Jacobin matrix with respect to x, which is given below