Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Physics Letters A 346 (2005) 315–320
PHYSICS LETTERS A
www.elsevier.com/locate/pla

Effects of periodic modulation on the Landau–Zener transition

Duan Suqing, Li-Bin Fu, Jie Liu, Xian-Geng Zhao

Institute of Applied Physics and Computational Mathematics, P.O. Box 8009 (28), 100088 Beijing, China

Received 2 June 2005; accepted 29 July 2005
Available online 15 August 2005
Communicated by J. Flouquet

Abstract

We study the quantum tunnelling of a two-level crossing system which extends the standard Landau–Zener model with applying a periodic modulation on its energy sweep. By directly integrating the time evolution operator we obtain the analytic expressions of tunnelling probability in the cases of high and low modulation frequency limit as well as in weak inter-level coupling limit. Our formula clarify the conditions for resonance occurrence, with the help of it we can readily manipulate the system in a desired way, say, to enhance or suppress the tunnelling probability effectively through adjusting the modulation properly.

PACS: 33.80.Be; 42.50.Md; 34.50.Rk; 75.40.Gb

Keywords: Landau–Zener tunnelling; Periodic modulation; Analytic solution

Avoided crossing of energy levels is a universal character for quantum nonintegrable systems where the break of system's symmetry leads to the splitting of degenerate energy levels forming a tiny energy gap. Around the avoided crossing point, the Landau–Zener (LZ) model provides an effective description for the tunnelling dynamics of the system with assuming that the energy bias of two levels undergoes a linear change with time [1]. LZ model is a rather general and fundamental model in quantum mechanics and has versatile applications in quantum chemistry [2], collision theory [3], and more recently, in the spin tunnelling of nanomagnets [4], Bose–Einstein condensates [5] and quantum computing [6].

There exist some extensions of the LZ model, including nonlinear LZ model [7], LZ problem with nonlinear energy sweep (square function of time) [8], LZ model with a fast noise mimic system's interaction with environment [9], to name only a few. On another aspect, there has been growing interest in the population dynamics of two-level systems under a periodic or quasi-periodic perturbation because of advances in the laser physics and fabrication techniques of mesoscopic systems. The probability of quantum tunnelling of an electron in a double-

E-mail addresses: duan_suqingd@mail.iapcm.ac.cn, duan_suqing@iapcm.ac.cn (D. Suqing).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2005.07.086

well potential is shown to be successively controlled by applying a periodic modulation of the relative energy of the wells [10]. As is mentioned above, the LZ model is a rather general and fundamental model in quantum mechanics, investigating its response to the external periodic modulation is a topics of great interest. Recently, Kayanuma and Mizaumoto [11] have discussed this issue. They emphasize on the population dynamics and find a series of step-like changes on the temporal evolution of level populations with using a transfer-matrix formalism.

In this Letter, we study this problem in a different way, i.e. directly integrating the time evolution operator. We concentrate on the total tunnelling probability, i.e. the population at final time, which is most concerned in practical situation. By directly integrating the evolution operator of Hamiltonian describing the LZ model with a periodic modulation, we are able to obtain the analytical expressions of total tunnelling probability. We find that, in both low and high frequency limits of periodic modulation the probability takes the same exponential form as the LZ formula but the energy gap is renormalized by modulation parameters; In the weak inter-level coupling limit, it takes a sinusoidal-like function predicting some resonance structures. Compared with the transfer-matrix method [11], our method is rather straight and our result is more concise and shows a better agreement with the numerical simulations.

We consider a time-dependent external electric field $\vec{E}(t)$, applied to the system modulating the energy bias between two states, denoted by $|1\rangle$ and $|2\rangle$. The operator for the electric-dipole moment $\vec{\mu}$ may be written as follows provided that the charge distribution of $|1\rangle$ and $|2\rangle$ is well separated,

$$\vec{\mu} = \frac{e\vec{a}}{2}\big(|1\rangle\langle 1| - |2\rangle\langle 2|\big), \tag{1}$$

where e is the electric charge of the electron and $\vec{a}$ is the vector connecting the two equilibrium points of the wells. The perturbation energy is then written as $-\vec{\mu}\cdot\vec{E}(t)$. We consider a case where $\vec{E}(t)$ is composed of two components, $\vec{E}(t) = \vec{E}_0 t + \vec{E}_1 \cos\omega t$. The model Hamiltonian then takes the form,

$$H(t) = \gamma\big(|1\rangle\langle 1| - |2\rangle\langle 2|\big) + \Delta\big(|1\rangle\langle 2| + |2\rangle\langle 1|\big), \tag{2}$$

where $\gamma = \frac{1}{2}(vt - A\cos\omega t)$, $v \equiv -e\vec{a}\cdot\vec{E}_0$, $A \equiv e\vec{a}\cdot\vec{E}_1$. Here the parameter γ denotes the energy bias between the two states.

The energy spectra of the system, as a function of the parameter γ, can be readily obtained by diagonalizing the above Hamiltonian. They show two straight lines of $\pm\gamma$ for asymptotic large energy bias, and an avoided crossing at the origin with the energy gap of Δ. Initially provided the energy bias is negative infinity, the two states is decoupled and both $|1\rangle$ and $|2\rangle$ are eigenstates. We suppose our particle populates on the lower level, i.e. $|1\rangle$ state. As we increase the energy bias from negative infinity to positive infinity, the particle has probability to tunnel to upper level due to quantum resonance occurred near avoided crossing point. Clearly the tunnelling process as well as the total transition probability at final time strongly depends on the way how we change the energy bias. The most simple way is the linear change with time, which gives the standard LZ model. In our case, we put a periodic modulation on the linear energy sweep, want to see how the periodic perturbation affects the quantum tunnelling. Here we mainly consider the total tunnelling probability, for it is most interest in physics.

For the state vector given by (we put $\hbar = 1$ throughout this Letter)

$$\big|\psi(t)\big\rangle = C_1(t)\exp\big\{-ivt^2/4 + i(A/2\omega)\sin\omega t\big\}|1\rangle + C_2(t)\exp\big\{ivt^2/4 - i(A/2\omega)\sin\omega t\big\}|2\rangle, \tag{3}$$

the Schrödinger equation is written as

$$i\frac{d}{dt}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \big[X(t)\sigma_x + Y(t)\sigma_y\big]\begin{pmatrix} C_1 \\ C_2 \end{pmatrix}, \tag{4}$$

where σ_x and σ_y are the Pauli matrices, and $X(t)$ and $Y(t)$ are defined as

$$X(t) = \Delta\cos\left(\frac{v}{2}t^2 - \frac{A}{\omega}\sin\omega t\right), \qquad Y(t) = -\Delta\sin\left(\frac{v}{2}t^2 - \frac{A}{\omega}\sin\omega t\right). \tag{5}$$