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Improving the time efficiency of the Fourier synthesis method for slice selection in magnetic resonance imaging

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ABSTRACT

The design of slice selective pulses for magnetic resonance imaging can be cast as an optimal control problem. The Fourier synthesis method is an existing approach to solve these optimal control problems. In this method the gradient field as well as the excitation field are switched rapidly and their amplitudes are calculated based on a Fourier series expansion. Here, we provide a novel insight into the Fourier synthesis method via representing the Bloch equation in spherical coordinates. Based on the spherical Bloch equation, we propose an alternative sequence of pulses that can be used for slice selection which is more time efficient compared to the original method. Simulation results demonstrate that while the performance of both methods is approximately the same, the required time for the proposed sequence of pulses is half of the original sequence of pulses. Furthermore, the slice selectivity of both sequences of pulses changes with radio frequency field inhomogeneities in a similar way. We also introduce a measure, referred to as gradient complexity, to compare the performance of both sequences of pulses. This measure indicates that for a desired level of uniformity in the excited slice, the gradient complexity for the proposed sequence of pulses is less than the original sequence.

Introduction

Magnetic resonance imaging (MRI) has become one of the most important medical imaging diagnostic tools available to physicians [1]. The basis of MRI is a phenomenon known as nuclear magnetic resonance (NMR). It is related to the way in which elementary particles such as protons interact with external magnetic fields having static and oscillating components. In MRI, it is possible to selectively excite the spins in a thin slice of the object by applying gradient fields [2]. To select a slice in a desired direction, a gradient field must be applied in that direction and a Radio Frequency (RF) excitation pulse with a limited bandwidth should be applied perpendicular to the gradient direction [2].

Many methods have been proposed to solve the selective excitation problem for adiabatic and non-adiabatic passages [3–9]. These methods are based on approximate solutions to the Bloch equation governing the spin dynamics in MRI, referred to as the Bloch equation [10–12], or are generated from computer simulations that predict the bulk magnetisation response to different excitation patterns [13,14]. Fourier analysis of the Bloch equation can be used to find a selective pulse rotating the bulk magnetisation less than $\pi/2$, see [10] and [15]. In this method, it is assumed that the Bloch equation behaves linearly for small tip angles.

The design of better pulses requires application of optimal control theory [16–19]. In [16] a mathematical basis for RF pulse design, an efficient algorithm to find the optimal pulse is provided. An optimal pulse is defined as the pulse that steers the magnetisation from the initial state, closest to the desired final state in a fixed amount of time. Ensemble controllability of the Bloch equation has been studied in [20,21].

The Shinnar–Le Roux (SLR) approach, which is ubiquitously used in designing pulses for MRI machines, is a recursive algorithm for finding the optimal pulse for a given selective excitation pattern [17]. In this approach the applied pulse is discretised to several rectangular pulses and the effect on the bulk magnetisation is calculated analytically at each step. As a result, the problem of finding selective pulses is reduced to the design of two polynomials. In this case, a selective RF pulse can be calculated through solving finite impulse response (FIR) filters. A number of variants of the SLR technique for designing selective pulses are available in the MRI literature [22–25].

Moore et al. in [5,7] have revisited the slice selection problem in the presence of the RF and the static field inhomogeneities using composite pulses. Their approach has two major steps. First, they optimise a non-selective composite pulse to minimise the effect of field inhomogeneities. In the second step, they use Gaussian or sinc sub-pulses with time varying gradient fields to enhance the slice

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selectivity of the overall sequence of pulses. The uniformity of the bulk magnetisation flip angle has been improved in a 7 Tesla human scanner in the presence of field inhomogeneities.

The Fourier synthesis method is an alternative technique to solve the optimal control problem for slice selection and RF field inhomogeneity suppression [26–30]. The gradient field as well as the excitation field are switched rapidly to shift the position of an ensemble of spins gradually. The magnitudes and duration of the applied fields are calculated based on a Fourier series expansion of the Bloch equation.

In this paper, we propose a novel sequence of pulses for the Fourier synthesis method that considerably improves the time efficiency of this technique while preserving its efficiency in terms of the selected slice quality. Mathematical induction is employed to form a proof of the Fourier synthesis method in the spherical coordinates. Numerical simulation for both sequences of pulses is compared in this paper. Simulation results indicate that the slice selectivity of both sequences of pulses behaves similarly in the presence of RF field inhomogeneities.

The Bloch equation

The behaviour of an ensemble of spins at a classical level in the presence of external magnetic fields may be described by the Bloch equation [31]. The Bloch equation in the classical rotating frame of reference whose transverse plane is rotating clock-wise at the Larmor frequency of the static field, ω_0, is written as

$$\begin{bmatrix}\dot{M}_{x'}\\ \dot{M}_{y'}\\ \dot{M}_{z'}\end{bmatrix}=\begin{bmatrix}-\dfrac{1}{T_2(\mathbf{r})} & \Delta\omega_0(\mathbf{r},t) & v(t)\\ -\Delta\omega_0(\mathbf{r},t) & -\dfrac{1}{T_2(\mathbf{r})} & u(t)\\ -v(t) & -u(t) & -\dfrac{1}{T_1(\mathbf{r})}\end{bmatrix}\begin{bmatrix}M_{x'}\\ M_{y'}\\ M_{z'}\end{bmatrix}+\frac{1}{T_1(r)}\begin{bmatrix}0\\ 0\\ M_0\end{bmatrix}, \tag{1}$$

where

$$u(t)\equiv\omega_{x'}(t)=\gamma B_1^e(t)\cos\phi_e, \tag{2a}$$

$$v(t)\equiv\omega_{y'}(t)=\gamma B_1^e(t)\sin\phi_e, \tag{2b}$$

and

$$\Delta\omega_0(\mathbf{r},t)=(\omega_0-\omega_{\mathrm{rf}})+\gamma\delta B_0+\gamma\mathbf{G}_{\mathbf{r}}(t)\cdot\mathbf{r}=\Delta\omega_{\mathrm{off}}+\delta\omega_0+\Delta\omega(\mathbf{r},t). \tag{3}$$

The parameters of the Bloch equation are summarised in Table 1. **r** is a vector representing position.

If the excitation field is initially applied in the x-direction then $\phi_e=0$, and we may write

$$u(t)\equiv\omega_{x'}(t)\equiv\omega_1(t)=\gamma B_1^e(t),\quad v(t)\equiv\omega_{y'}(t)=0. \tag{4}$$

Here, $\omega_1(t)$ is referred to as the Rabi frequency [32–34].

Control of a spin system by the Fourier synthesis method

For a short period of excitation (less than one time constant ($t\ll T_2(\mathbf{r})$), it is possible to ignore relaxation terms [35] and approximate the magnetic resonance phenomenon in the classical rotating frame of reference with the following equation:

$$\begin{bmatrix}\dot{M}_{x'}\\ \dot{M}_{y'}\\ \dot{M}_{z'}\end{bmatrix}=\begin{bmatrix}0 & \Delta\omega_0(\mathbf{r},t) & v(t)\\ -\Delta\omega_0(\mathbf{r},t) & 0 & u(t)\\ -v(t) & -u(t) & 0\end{bmatrix}\begin{bmatrix}M_{x'}\\ M_{y'}\\ M_{z'}\end{bmatrix}, \tag{5}$$

Table 1
Description of the Bloch equation parameters.

Parameter	Description
$\mathbf{M}(\mathbf{r},t)$	Magnetisation vector in laboratory frame of reference
$\mathbf{M}'(\mathbf{r},t)$	Magnetisation vector in rotating frame of reference
M_0	Bulk magnetisation magnitude at thermal equilibrium
$T_1(\mathbf{r})$	Longitudinal relaxation time constant
$T_2(\mathbf{r})$	Transverse relaxation time constant
γ	Gyromagnetic ratio
$\mathbf{B}_0$	External static field applied in the z-direction
$\mathbf{B}_1$	Excitation or rotating field applied in the xy-plane
B_1^e	Envelope of the excitation field
ϕ_e	Initial phase of the rotating field
$\mathbf{G}_{\mathbf{r}}(t)$	Gradient field
$\omega_0=\gamma B_0$	Larmor frequency of the static field
ω_{rf}	Oscillating frequency of the rotating field
$\omega_1(t)$	Rabi frequency
$\Delta\omega_{\mathrm{off}}$	Off-resonance excitation
$\delta\omega_0=\gamma\delta B_0$	Deviation from the Larmor frequency as a result of the static field imperfection and the tiny fields induced by the object under study
$\Delta\omega(\mathbf{r},t)$	Space-dependent frequency generated by gradient fields
$\Delta\omega_0(\mathbf{r},t)=\Delta\omega_{\mathrm{off}}+\delta\omega_0+\Delta\omega(\mathbf{r},t)$	Deviation from the Larmor frequency caused by all possible sources over space and time

where $u(t)$ and $v(t)$ are defined in Eq. (2). The above equation may be written as

$$\begin{bmatrix}\dot{M}_{x'}\\ \dot{M}_{y'}\\ \dot{M}_{z'}\end{bmatrix}=-(u(t)\Omega_{x'}-v(t)\Omega_{y'}-\Delta\omega_0(\mathbf{r},t)\Omega_{z'})\begin{bmatrix}M_{x'}\\ M_{y'}\\ M_{z'}\end{bmatrix}, \tag{6}$$

in which

$$\Omega_{x'}=\begin{bmatrix}0&0&0\\0&0&-1\\0&1&0\end{bmatrix},\quad \Omega_{y'}=\begin{bmatrix}0&0&1\\0&0&0\\-1&0&0\end{bmatrix},\quad \Omega_{z'}=\begin{bmatrix}0&1&0\\-1&0&0\\0&0&0\end{bmatrix}, \tag{7}$$

and we now assume that $\Delta\omega_0=\Delta\omega(\mathbf{r},t)=\Delta\omega(z,t)$ as only gradient in z-direction is applied and it dominates the $\Delta\omega_{\mathrm{off}}$ and $\delta\omega_0$ which are negligible.

Eq. (6) clearly indicates that the excitation and inhomogeneities cause the magnetisation vector to rotate about an axis.[1] If gradient fields are superimposed on the main static field then only the $\mathbf{G}_r(t)\cdot\mathbf{r}$ part of $\Delta\omega_0(\mathbf{r},t)$ is taken into account. Given that the gradient field is applied in the z-direction, the objective is to find controls, $u(t)$ and $v(t)$, that can drive the bulk magnetisation to the desired slice profile such that

$$J=\int_z\left\|\mathbf{M}_f(T_f,u_{[0,T_f]},v_{[0,T_f]},z)-\mathbf{M}_d(z)\right\|^2dz \tag{8}$$

is minimised. In the above equation, $\mathbf{M}_f$ is the final state after the pulse has been turned off at $t=T_f$ and $\mathbf{M}_d$ represents the desired state.

Review of the Fourier synthesis method for slice selection in Cartesian coordinates

The Fourier synthesis method may be used to solve the optimisation problem indicated by Eq. (8). In this technique during

[1] $\exp(\alpha\Omega_{x'})$, $\exp(\beta\Omega_{y'})$, and $\exp(\gamma\Omega_{z'})$ generate rotation matrices about x', y', and z' axes, respectively.