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Technical Notes

# Spline modelling electron insert factors using routine measurements

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ABSTRACT

There are many methods available to predict electron output factors; however, many centres still measure the factors for each irregular electron field. Creating an electron output factor prediction model that approaches measurement accuracy – but uses already available data and is simple to implement – would be advantageous in the clinical setting. This work presents an empirical spline model for output factor prediction that requires only the measured factors for arbitrary insert shapes. Equivalent ellipses of the insert shapes are determined and then parameterised by width and ratio of perimeter to area. This takes into account changes in lateral scatter, bremsstrahlung produced in the insert material, and scatter from the edge of the insert. Agreement between prediction and measurement for the 12 MeV validation data had an uncertainty of 0.4% (1SD). The maximum recorded deviation between measurement and prediction over the range of energies was 1.0%. The validation methodology showed that one may expect an approximate uncertainty of 0.5% (1SD) when as little as eight data points are used. The level of accuracy combined with the ease with which this model can be generated demonstrates its suitability for clinical use. Implementation of this method is freely available for download at https://github.com/SimonBiggs/electronfactors.

## Introduction

Electron beams are often used for the treatment of skin tumours, head and neck cancers, and breast boosts. The dose delivered by an electron beam is primarily dependent upon beam energy, patient specific field shape, SSD, and collimator design as well as patient anatomy. The specific shape used in the final aperture collimation of the treatment applicator is often unique for any given patient and will here be referred to as the insert as per AAPM TG 71 [1]. For our purposes here the portion of the output factor that is dependent on the insert will be called the insert factor. Even though there are methods in the literature for modelling the insert factor, in many centres it is often directly measured [1].

The modelling methods available range in complexity, accuracy, and resources required to implement. Historical methods such as the square root method are still in use and, when compared to measurement, can achieve uncertainties (represented to 1SD) as low as 2% [2]. Analytical modelling methods built atop of the Fermi–Eyges pencil beam model such as the lateral build-up method have achieved uncertainties as low as 1% [3]. More sophisticated Monte Carlo methods have successfully predicted insert factors to an uncertainty as low as 1% [4]. Empirical models such as the sector integration method [5] have shown promise with a particular enhanced method achieving uncertainties as low as 0.6% [6]; however, it requires many measurements of circular fields.

To create a model that may be widely adopted in the clinical setting it must be easy to implement and have an uncertainty approaching that already available via measurement. If the use of electron fields is infrequent or high accuracy is desired then direct measurement for each irregular insert may be preferred over the implementation of a numerical method or the measurement of specific inserts for an empirical method. Most clinics have vast amounts of data currently available to them. Creating a model that builds upon these already collected data would be beneficial. An empirical model via parameterising insert shapes is a promising way of achieving these aims.

There are a number of parameters that can be constructed to characterise the dependence of delivered dose on insert shape. For the shapes where sections of the insert begin to reduce lateral scatter to the point of maximum dose, the smallest dimension of the shape becomes an important factor for prediction. Examples of models taking into account this smallest dimension are bivariate polynomial fits [7], equivalent radii methods, or building a shape out of sectors with given radii such as the sector integration method [5]. However the lateral scatter of the insert is not the only physical effect

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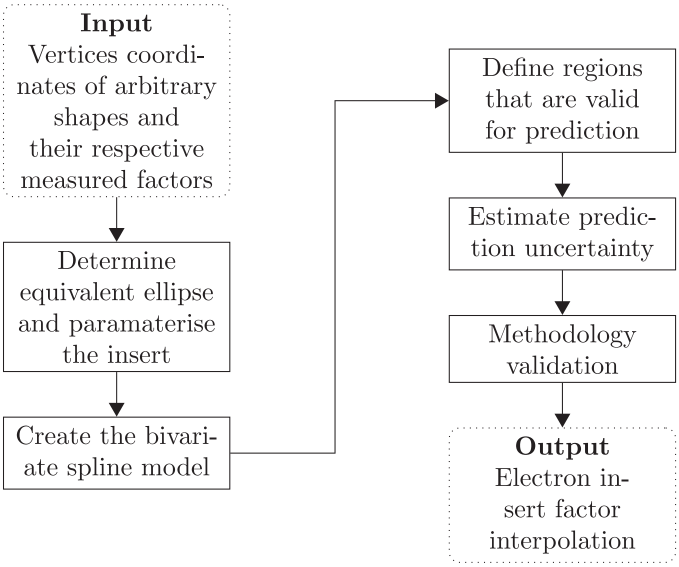

**Figure 1.** Flow chart of the equivalent ellipse spline model methodology.

on the insert factor. Bremsstrahlung produced in the insert material, transmission, and scatter off the internal surface of the insert can have a combined effect of 2–4% on the insert factor [8]. The larger the area of the insert aperture, the lower the contribution to dose is from the insert material, however the insert scatter effect increases with collimating surface, which can be approximated as having a dependence on shape perimeter. These physical effects therefore may be aptly parameterised by the area and the perimeter of the insert shape. The work of Nair et al. [9] demonstrates the feasibility of such parameterisation, and expansion on these ideas is a promising area of investigation.

Presented here is an empirical bivariate model based upon the two parameters width and the ratio of perimeter to area. The model takes arbitrary insert shapes as input data and interpolates between them (see Fig. 1). Each shape is simplified to be represented by an equivalent ellipse, determined so that the width and area are equal. Modelling of insert factors is done using a smoothing bivariate spline model [10] and the Scientific Stack for Python [11] using the Anaconda Python 3.4 distribution.

## Methods

### *Insert factor measurements*

Insert factors were measured in RW3 with an Advanced Markus on an Elekta Agility linac. Definitions of electron output factors given here are as per AAPM TG 25, defined as the ratio of dose per monitor unit at $d_{max}$ [12]. When the depth of maximum dose is shifted from the reference depth this depth was searched for to a 1 mm resolution. All depth differences took into account stopping power ratio corrections as per the protocol in IAEA TRS 398 [13]. The depth searching process was aided by automated relative dose plotting by a program written in python which performed the ionisation to dose conversion.

For the purpose of methodology validation a large data set of 42 shapes was created. Of these, 24 shapes were pulled from previously treated clinical shapes stored within the treatment planning system. Supplementing the clinical shapes were 18 standard shapes, 7 circles and 11 ellipses. The circles ranged in diameter from 3.2 cm to 9.5 cm at 100 cm SSD. The aspect ratios of the ellipses varied between 1.6 and 4.3 with the highest aspect ratio ellipse being 3.2 cm × 13.6 cm. These 42 insert shapes were measured using 12 MeV electrons on the 10 cm applicator. Data were also collected to confirm the methodology for the remaining energies, 6, 9, 15, and 18 MeV with shapes over the clinical range of the 10 cm applicator. These consisted of the same set of five circles and three ellipses. The five circles ranged in diameter from 5.0 to 9.5 cm at 100 cm SSD. The highest aspect ratio ellipse chosen was 5.3 cm × 12.4 cm. These data were combined with previously measured clinical measurements ranging in number from four to seven depending on energy.

### *Equivalent ellipse parameterisation*

The methodology proposed here for finding the equivalent ellipse is similar to the method for an equivalent rectangle as proposed by Hogstrom et al. reproduced in AAPM TG 71 [1].

The width of the ellipse was chosen to produce equivalent loss of lateral scatter effects between the insert and the ellipse. It is defined as the diameter of the largest circle fully enclosed by the insert shape. The circle meeting these conditions was found using the Basin-Hopping global optimiser [14], the BFGS local optimiser [15], and the Shapely geometry python module. This method of parameterising loss of lateral scatter assumes that the loss of scatter is primarily from the minor axis of the shape. If lateral scatter is being lost in the major axis of the shape discretion is required to determine if a similar loss occurs in the resulting equivalent ellipse.

The length of the ellipse is chosen so that there is similar bremsstrahlung production in the shielding between the ellipse and the insert. This is done by choosing the length so that the area of the ellipse is equal to that of the insert shape. This results in a similar volume of shielding for both the original insert and the equivalent ellipse. In this process the perimeter of the ellipse no longer remains equal to that of the original insert, however when comparing the inserts and their corresponding equivalent ellipses the difference in the resulting insert factor prediction was small.

The final parameters used for modelling were width and the ratio of perimeter to area defined using the equivalent ellipse. To calculate the perimeter (P) of the ellipse given its width (w) and length (l) Ramanujan's approximation was used as given in Eq. (1):

$$P \approx \frac{\pi}{2}\left[3(w+l)-\sqrt{(3w+l)(3l+w)}\right] \tag{1}$$

### *Bivariate spline model*

Fitting was achieved using the smoothing bivariate spline [10] class packaged within SciPy [11]. This was used to create a fit for the measured insert factors against the two parameters width and the ratio of perimeter to area. It was found that the required spline orders were no more than two in the width dimension, and one in the ratio of perimeter to area dimension. The spline order was kept as low as possible to reduce the impact of outliers on the fit and to allow for greater coverage of the valid region. Due to this choice of spline order, combined with the requirement to remove a data point for uncertainty estimation, the minimum number of unique equivalent ellipses required to create this model is eight. The smoothing factor used within the spline class was chosen to be equal to the number of input data points as recommended by the SciPy online documentation. The spline bounding box was expanded to include the requested point of prediction, this was important when extrapolating outside the original spline bounding box. The output of the spline model is an insert factor prediction function, $f(w, {}^{P}\!/_{A})$, taking the inputs width ($w$) and ratio of perimeter to area (${}^{P}\!/_{A}$) and returning an insert factor prediction.