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On the parametrization of lateral dose profiles in proton radiation therapy

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Purpose: The accurate evaluation of the lateral dose profile is an important issue in the field of proton radiation therapy. The beam spread, due to Multiple Coulomb Scattering (MCS), is described by the Molière's theory. To take into account also the contribution of nuclear interactions, modern Treatment Planning Systems (TPSs) generally approximate the dose profiles by a sum of Gaussian functions. In this paper we have compared different parametrizations for the lateral dose profile of protons in water for therapeutical energies: the goal is to improve the performances of the actual treatment planning.
Methods: We have simulated typical dose profiles at the CNAO (Centro Nazionale di Adroterapia Oncologica) beamline with the FLUKA code and validated them with data taken at CNAO considering different energies and depths. We then performed best fits of the lateral dose profiles for different functions using ROOT and MINUIT.
Results: The accuracy of the best fits was analyzed by evaluating the reduced χ^2, the number of free parameters of the functions and the calculation time. The best results were obtained with the triple Gaussian and double Gaussian Lorentz–Cauchy functions which have 6 parameters, but good results were also obtained with the so called Gauss–Rutherford function which has only 4 parameters.
Conclusions: The comparison of the studied functions with accurate and validated Monte Carlo calculations and with experimental data from CNAO lead us to propose an original parametrization, the Gauss–Rutherford function, to describe the lateral dose profiles of proton beams.

1. Introduction

Proton therapy has become a successful treatment for radioresistant tumors and the number of centers with proton beams is increasing at a steady pace worldwide, as testified by the 2014 NuPECC report [1]. Following this trend the demand for better accuracy in the dose calculation is also increasing: the study of dose deposition in tissue is a very complex task and in principle can be solved with the Monte Carlo (MC) technique. Nonetheless this solution cannot yet be adopted for treatment planning because, despite its reliability, it is not sufficiently fast for the clinical applications [2]. In particular, the parametrization of the lateral dose profile is an important issue in this context: the Multiple Coulomb Scattering (MCS) of the beam particles, described by the Molière's theory [3], accounts only for electromagnetic interactions between the beam particles and the irradiated materials while the global transverse dose profile, which also accounts for the contribution of nuclear interactions, is generally approximated by a sum of Gaussian functions by most of the modern Treatment Planning Systems (TPSs) [4]. This allows achieving a good compromise between calculation accuracy and computing efficiency: nevertheless, a great effort is currently devoted to enhance the accuracy of the calculations for proton treatments without increasing the computation time. In this frame, correct parametrization of the non-Gaussian tails of the dose distribution, to account for all the physical contributions (electromagnetic and nuclear), is necessary. In this work, after a short

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review of the MCS theory to justify the use of Gaussian functions starting from a typical TPS parametrization with two Gaussians, we show how to improve the description of the tails of the dose distributions by using alternative functions. We compare the results with MC simulation and experimental data in the case for proton beams at energies of clinical relevance and discuss the computational accuracy and efficiency of the proposed solutions.

1.1. Review of multiple scattering theory

The scattering of charged particles in materials is described by the well established Molière's theory that allows calculating the deflections of the primary particle trajectories from the original direction in terms of a spatial angle θ. The solution is independent of the azimuthal angle φ and is usually expressed in terms of the projected angles θ_x and θ_y (Fig. 1).

Molière has found a probability distribution function for θ that is a solution of the transport equation and that well describes the experimental data [3]:

$$f(\theta)\theta d\theta = f_M(\theta)d(cos\theta)d\phi/2\pi \tag{1}$$

and, by using the approximation $|d(cos\theta)| = sin\theta\, \mathrm{d}\theta \approx \theta\, \mathrm{d}\theta$, can be written as the sum of three terms:

$$f_M(\theta) = \frac{1}{2\pi}\frac{1}{2\theta_M^2}\left[f^0(\theta') + \frac{f^1(\theta')}{B}\right) \tag{2}$$

where θ_M is the characteristic multiple scattering angle of the target, $\theta' = \theta/(\sqrt{2}\theta_M)$ is the reduced angle and B is related to the logarithm of the collisions effective number in the target. This expression is at the heart of the multi-Gaussian parametrization of the lateral dose profiles that is found in literature [4,5] and is often used by many TPS codes, due to the presence of a Gaussian core and of the additional terms fully describing the tails of the dose distribution. The functions $f^k(\theta')$ are given by the general expression:

$$f^k(\theta') = \frac{1}{n!}\int_0^\infty yJ_0(\theta' y)e^{-y^2/4}\left(\frac{y^2}{4}ln\frac{y^2}{4}\right)^k \mathrm{d}y \tag{3}$$

where J_0 is the Bessel function of the first kind of order 0. In terms of $x = \theta'^2$, these functions can be explicitly written as [6,7].

$$\begin{cases} f^0(x) = 2e^{-x} \\ f^1(x) = 2e^{-x}(x-1)\left[\overline{Ei}(x) - lnx\right] - 2\left(1 - 2e^{-x}\right) \\ f^2(x) = e^{-x}\left(\left[\psi^2(2) + \psi(2)\right]\left(x^2 - 4x + 2\right) + 4\int_0^1 y^{-3}dy[lny/(1-y) - \psi(2)] \times \left[\left(1-y^2\right)e^{xy} - 1 - (x-2)y - \left(x^2/2 - 2x + 1\right)y^2\right]\right) \end{cases}$$

where $\psi(n) = \mathrm{d}\{ln\Gamma(n+1)\}/\mathrm{d}n$ is the Digamma function and $Ei(x)$ is the Exponential Integral function. The first term is a Gaussian function that represents the core of the distribution, while the extra terms account for the tails of the distribution that are non-Gaussian: they can be evaluated by numerical integration or by using mathematical tables [8]. In physical terms, the core of the distribution takes into account only the pure electromagnetic part of the multiple scattering process (Molière's theory), while the tails in addition describe also the nuclear interactions of the proton beam: these play an important role in proton therapy due to the inelastic reactions and cannot be neglected [9].

The electromagnetic contribution can be calculated analytically by the full Molière's theory, but is often approximated by a single Gaussian function corresponding to the $f^0(\theta')$ term above, as in the Highland parametrization [10]. More general parametrizations are based on a sum of two or more Gaussians or other functions that try to take into account the extra terms [11–14].

The analytic calculation of the nuclear contributions is a too difficult task and a MC approach is still very time consuming with the present computing technology [2]: therefore many empirical parametrizations for the nuclear halo are available in literature. Among these it is worth mentioning the approach of Soukup et al. [12] that parametrizes the nuclear halo tails with a modified Cauchy–Lorentz function, the work of Li et al. [13] and the recent review by Gottschalk [14] that proposes a very detailed model with 25 parameters.

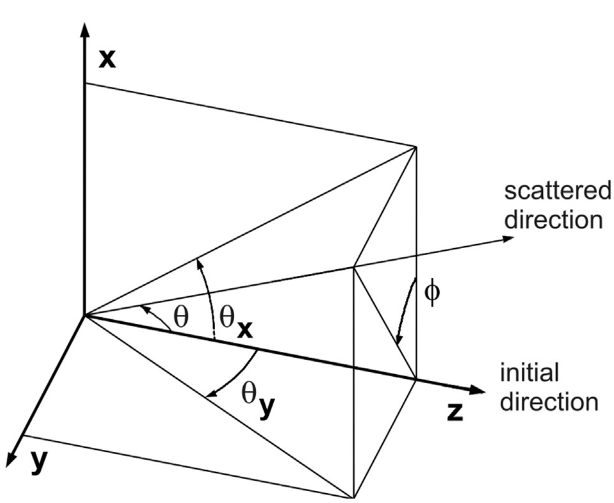

Figure 1. Polar and azimuthal angles used in Molière's theory.

2. Materials and methods

2.1. Parametrizations

In this paper we described the search for a model which can be adopted by a TPS to be not only fast for dose calculation but also accurate in estimating the lateral dose released by proton beams in the clinically relevant energy range.

The theory of multiple scattering can be easily summarized assuming that the form of the lateral beam profile comes from the combination of two processes: the electromagnetic interactions, described by Molière's theory and giving contributions to the core and to the tails of the distribution, and the nuclear interactions,