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Constructing rogue wave prototypes of nonlinear evolution equations via an extended tanh method

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ABSTRACT

In this paper, we show that the extended tanh method [*Z.S. Lü, H.Q. Zhang, (2003) [37]*] can be applied to construct exact breather-type solutions for nonlinear evolution equations, and those solutions might be used as rogue wave prototypes. As illustrative examples, we present exact solutions for the Boiti–Leon–Pempinelli equation and the $(2+1)-$dimensional breaking soliton equation respectively. The solution waves include rational rogue waves and non-rational rogue waves.

1. Introduction

In the past few decades, many efforts have been made to investigate various localized nonlinear waves such as the soliton [1], the breather [2], the compacton [3], and the dromion [4–6], etc., and these investigations in turn promoted the development of many useful mathematical tools [1,7–10]. Particularly, in recent years, some symbolic computation aided constructive algorithms have been proposed for seeking exact solutions of nonlinear evolution equations (NLEEs) [11–16].

Rogue waves are giant single waves appearing in the ocean "from nowhere" and disappear "without a trace" [17]. Due to its great destruction power, rogue waves have long been under the investigations of ocean scientists. In 2007, D.R. Solli et al. first reported the experimental observation of optical rogue waves [18]. Since then, rogue waves have been studied and applied extensively in different fields including atmosphere [19], plasma physics [20], Bose–Einstein condensate [21,22], microwave [23], superfluid [24], and even finance [25]. In reference [26], the main generating mechanisms of rogue waves in different physical contexts were elucidated and compared.

Generally, complex field and real field NLEEs have different physical origin which influences their nature. Even the same type of solution waves (for example kink solutions or anti-kink solutions) for the two types of NLEEs may have different travelling properties [27]. It is found that many $(1+1)-$dimensional complex field NLEEs possess rogue wave solutions. However, to the authors' knowledge, there are much less results on rogue wave solutions for real field, especially high dimensional real field NLEEs [28,29].

Currently, Darboux transformation and its generalizations, as well as algebra-geometric methods are the most commonly used methods for seeking rogue wave solutions of NLEEs [28,30–36]. In this paper, we will show that the extended tanh method [37] can be applied to construct exact breather-type solutions for NLEEs, and those solutions might be used as rogue wave prototypes. As illustrative examples, we consider the Boiti–Leon–Pempinelli (BLP) equation and the $(2+1)-$dimensional breaking soliton equation. The solutions we obtain include rational rogue waves and non-rational rogue waves.

2. Rogue wave prototypes for two nonlinear evolution equations

In this section, we take two nonlinear models as examples to illustrate how the extended tanh method can be applied

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to construct breather-type solutions (which can be served as rogue wave prototypes) for the NLEEs.

Example 1. We consider the following BLP equation

$$u_{ty} = (u^2 - u_x)_{xy} + 2\,v_{xx}$$

$$v_t = v_{xx} + 2\,(u\,v)_x, \tag{1}$$

which was first appeared in the reference [38] as a $(2+1)-$dimensional generalization of the sine- and

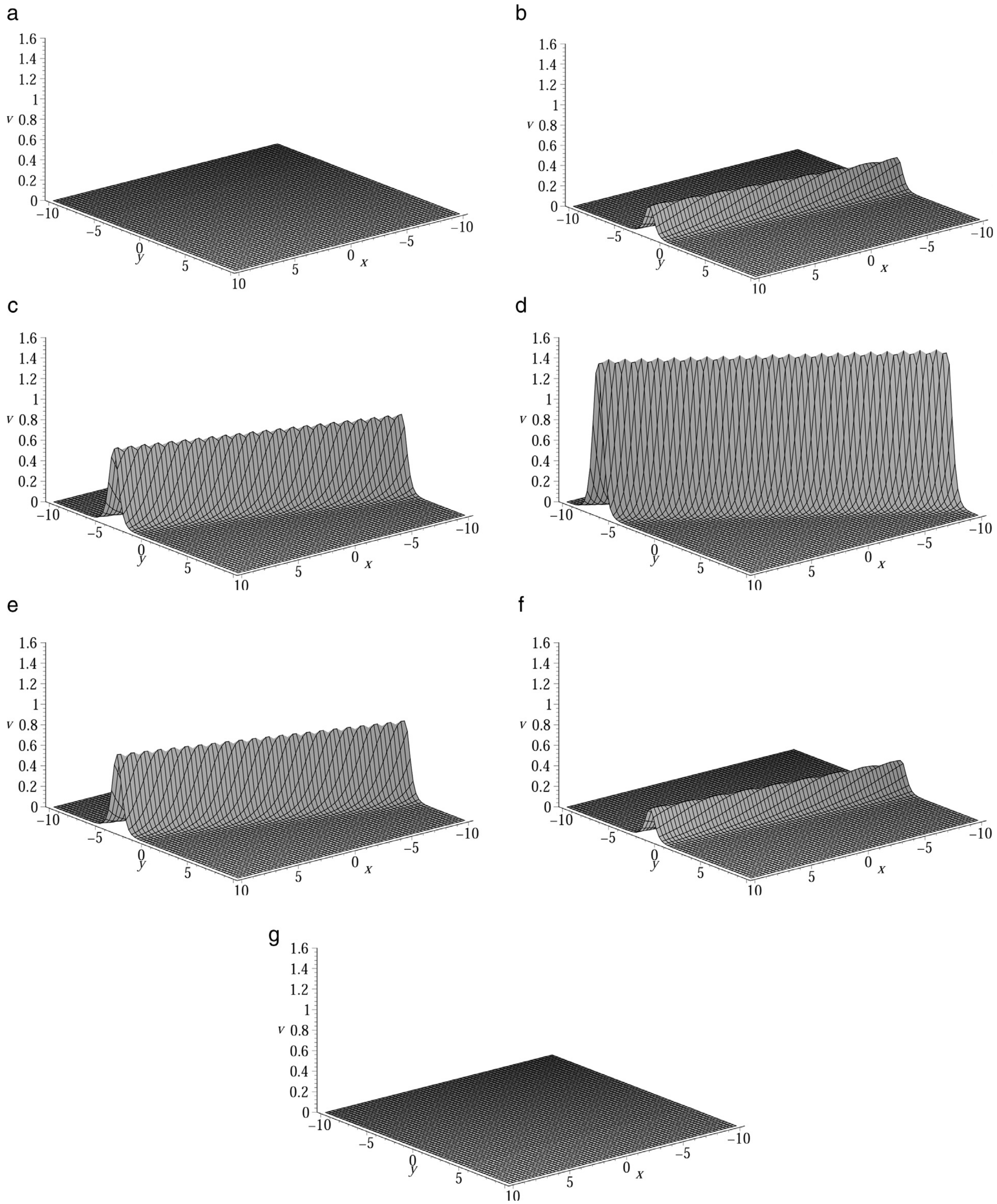

Fig. 1. Plots of v_1 with the parameter restriction (8) at times $(a)\ t = -15$, $(b)\ t = -1$, $(c)\ t = -0.5$, $(d)\ t = 0$, $(e)\ t = 0.5$, $(f)\ t = 1$, $(g)\ t = 15$.