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A new hybrid algorithm based on chaotic maps for solving systems of nonlinear equations

J. Alikhani Koupaei [a,b,*], S.M.M. Hosseini [a,c]

[a] Department of Mathematics, Yazd University, P.O Box 89195-74, Yazd, Iran
[b] Department of Mathematics, Payame Noor University, P.O Box 19395-3697, Tehran, Iran
[c] Department of Applied Mathematics, Shahid Bahonar University of Kerman, Kerman, Iran

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ABSTRACT

This study attempts to present a new hybrid algorithm combining the capabilities of chaotic maps and the golden section search method to solve system of nonlinear equations. The proposed algorithm consists of two main stages. In the first stage, system of nonlinear equations is transformed into an unconstrained optimization problem. In the second stage, a bipartite experimental procedure is introduced (1) Chaotic reduction explores a sub-space to satisfy unimodal condition of the problem utilizing the chaotic maps as a global search. (2) The n-dimension golden section search method (GSS) is applied as a local search over the achieved search space to exploit the optimization problem. In order to study the performance of the proposed algorithm, eleven well-known problems are employed. The findings revealed that the approach is potentially capable of solving various types of systems of nonlinear equations with great precision. Moreover, the numerical results revealed that our model is an effective and efficient method in comparison with some state-of-the-art algorithms.

1. Introduction

Systems of nonlinear equations (SNLE) are employed in many real-world applications such as chemistry [1], economics [2], physics [3–6] and engineering [7–9]. These problems are mainly difficult to solve due to their intrinsic computational complexity as well as the existence of various solutions. There are different methods of solving these kinds of problems. Two main categories of these techniques called interval methods [10–20] and continuation methods [21,22]. Solving nonlinear set of equations with interval methods is guaranteed, but tended to be slow [23]. Continuation methods are more efficient for SNLE, which has not too high total degrees [23]. Newton-type methods could be a good selection for solving systems of nonlinear equations. However, their convergence is very sensitive to the choice of initial guess for the solution. Over the course of the past decades, a diverse range of methods and techniques have been developed to solve SNLE. Karr et al. [24] solved a system of nonlinear equations using a combination of the Genetic algorithm and Newton's method. The Genetic algorithms explore the space problem to find an initial solution, which improved using a good local search based on gradient method [24]. The HPSO algorithm used the advantages of PSO and Nelder–Mead simplex method (SM) to solve SNLE [25]. The hybrid PSO was applied to find an initial value of SM, which increased the performance of SM and PSO. In order to decrease the sensitivity of algorithms to initial guess of the solution, the combination of chaotic search and Newton-type methods was used. Latter, improves the performance of gradient based methods using the global search capabilities of chaotic optimization [26]. The CDPSO changed high-dimensional into

* Corresponding author at: Department of Mathematics, Yazd University, P.O Box 89195-74, Yazd, Iran. Tel.: +9803531232515.
E-mail addresses: verk500@yahoo.co.uk (J.A. Koupaei), hosse_m@yazd.ac.ir (S.M.M. Hosseini).

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low dimensional problem applying the conjugate direction algorithm (CD) into particle swarm optimization (CDPSO). The change has been made in order to overcome local optimum for solving nonlinear system of equations [27]. The PPSO algorithm presented a new updating rule for each particle in PSO algorithm [28]. In this algorithm, we need to set seven random parameters for a problem which decreases the robustness and efficiency of PPSO. In the same way, Abdollahi et al. [29] employed the adaptive imperialist competitive algorithm (ICA) for solving a nonlinear system of equations. However, the ICA algorithm falls in the local optimum and setting some initial parameters is required. Lately, Turgut et al. [30] introduced a novel chaotic quantum behaved PSO algorithm for solving these problems. Although this algorithm improves the effectiveness and robustness of QPSO [31] using chaotic maps, it often suffers from trapped in local optimum. Furthermore, it has high computational time. Song et al. [32] presented a generic transformation method based on multi-objective optimization of nonlinear equation systems which called MONES. Subsequently, the problem optimized by multi-objective evolutionary algorithms. However, the basic concept of evolutionary algorithm leads to some drawbacks and there still exist some obstacles in solving system of nonlinear equations.

Recently, there is a tendency towards chaos optimization methods. Chaotic behaviors are similar to randomness and they are untidy, although they are created by deterministic iteration formulas such as Logistic map, Chebyshev map and so on [33–36]. Chaotic optimization methods are based on the regularity of chaotic motion in the search space of the problem. Hence, the convergence criterion is satisfied. The empirical validation has revealed that the chaotic optimization methods have global search ability, and they are effective only in small feasible spaces [37].

The golden section search (GSS) algorithm is a practical technique to optimize a unimodal objective function without computing gradient of objective function. The GSS method is implemented based on the dividing search space. Although GSS has provided some important advantages such as high speed of convergence, easy implementation and guaranteed convergence, it has some basic constraints such as unimodal and 1-D objective functions [38]. The extended version of GSS has introduced to solve the dimensional limitation of GSS in *n*-dimension problems [38], but the algorithm cannot support the multimodal functions. In order to overcome these problems, we proposed a new practical algorithm combining the golden section search and chaotic maps to solve nonlinear systems of equations. Some algorithms suggest a novel technique for solving nonlinear systems of equations by converting the problem into an optimization problem. Let the form of SNLE be:

$$\begin{cases} f_1(x_1, x_2, \ldots, x_n) = 0, \\ f_2(x_1, x_2, \ldots, x_n) = 0, \\ \qquad \vdots \\ f_m(x_1, x_2, \ldots, x_n) = 0 \end{cases} \tag{1}$$

where $f_i : D \subset R^n \to R$, $D = [a_1, b_1] \times [a_2, b_2] \ldots \times [a_n, b_n] = [l, u]$, $i = 1, 2, \ldots, m$ are nonlinear equations and $x = (x_1, x_2, \ldots, x_n)$ is the unknown vector. The equations system (1) is transformed into an optimization problem as follows:

$$\min F(x) = \sum_{i=1}^{m} f_i^2(x), \ x \in D. \tag{2}$$

This problem is similar to the approach used in [28].

In this paper, we proposed a new perspective of multimodal *n*-dimension GSS for solving systems of nonlinear equations consisting of chaotic maps and n-D GSS methods which is called CG-SNLE. The present study aimed to balance the exploration and exploitation of the algorithm for solving SNLE problems. The algorithm uses the ergodicity of chaotic maps to reduce the searching space and to increase the efficiency of GSS method. The chaotic maps can satisfy unimodal condition for objective function. Thus, the golden section search method can be used to solve the SNLE problem on the new feasible space.

For this purpose, System (1) transforms into a kind of optimization problem (2) then a bipartite experimental procedure is used to solve the optimization problem. First, chaotic reduction is used to determine a small search space using chaotic concept to satisfy unimodal condition of problem (2) for the n-D GSS algorithm. Second, the n-D golden section search exploits the optimum solution over the obtained search space from chaotic reduction step. In the other hand, chaotic reduction step can be directly applied to handle the multimodal problems. In order to verify the effectiveness of CG-SNLE method, it has been integrated into n-D GSS to solve SNLE problems.

To study the performance of the proposed algorithm, some well-known nonlinear systems were employed. The findings demonstrated the superior performance of the CG-SNLE proposed for solving SNLE problems. The experimental results revealed that the proposed algorithm is an effective and efficient method in comparison with some state-of-the-art methods for solving SNLE problems.

The rest of this paper is set up as follows: Section 2 describes optimization algorithms such as basic chaos optimization algorithm (COA) and the golden section search algorithm. Section 3 introduces our CG-SNLE method for solving SNLE problems. Experimental and numerical results of some the well-known systems are discussed to demonstrate the operational superiority of the proposed algorithm in Section 4. Finally in the last section, the conclusions are presented.

2. Related works

In this section, we discuss some of the optimization algorithms such as the first carrier wave method (FCW) and the GSS algorithm which are used in the literature.

2.1. Chaotic optimizing methods

Chaos is a random-like behavior in a nonlinear and deterministic system which is dependent on initial conditions. Edward Lorenz has shown the first chaotic attractor in 1963. He has presented one of the most famous images of chaos theory that known as 'butterfly effect' [39]. The ideas of chaos theory have been used in a wide range of fields. Chaotic optimizing methods are new optimization techniques based on using numerical sequences created by chaotic maps. In