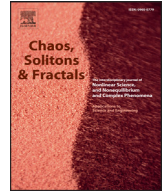


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Bifurcation analysis and Turing instability in a diffusive predator-prey model with herd behavior and hyperbolic mortality

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ABSTRACT

In this paper, we consider a predator-prey model with herd behavior and hyperbolic mortality subject to the homogeneous Neumann boundary condition. Firstly, we prove the existence and uniqueness of positive equilibrium for this model by analytical skills. Then we analyze the stability of the positive equilibrium, Turing instability, and the existence of Hopf, steady state bifurcations. Finally, by calculating the normal form on the center manifold, the formulas determining the direction and the stability of Hopf bifurcations are explicitly derived. Meanwhile, for the steady state bifurcation, the possibility of pitchfork bifurcation can be concluded by the normal form, which does also determine the stability of spatially inhomogeneous steady states. Furthermore, some numerical simulations to illustrate the theoretical analysis are also carried out and expand our theoretical results.

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1. Introduction

Since the pioneering works of Lotka and Volterra [29,30], numerous mathematical models have been proposed to study the relation between predator population and prey population. Due to the importance of such models in ecology, predator-prey models have been intensively studied and will continue to be one of the dominant themes in the area. Predator-prey models are basic differential equation models for describing the interactions between two species, and have been widely studied by researchers in mathematics and ecology.

It is well known that there are many factors which affect population dynamics in predator-prey models. One crucial component of predator-prey relationships is predator-prey

interaction, which can be classified into many different types, such as Holling I–IV types, Hassell–Varley type, Beddington–DeAngelis type, Crowley–Martin type, and so on. Moreover, we have noted that cyclical interactions in structured populations are another type predator-prey interactions, which are rightfully acquiring a central role in the study of evolutionary processes, not just in the realm of predator-prey modeling, but also in evolutionary game theory [19]. In recent years, cyclical interactions emerging in multi-species predator-prey system and in evolutionary settings relevant to public goods have been investigated widely. Examples include that cyclical interactions with alliance-specific heterogeneous invasion rates in [14], noise-guided evolution in [13], where it was shown that defensive alliances can emerge if the competing strategies are more than 3, and public goods game with spontaneously emerging cyclic dominance, see [22–25]. For the potential applicability of the proposed theory on cyclic dominance extending to the physics of social systems and to statistical mechanics of evolutionary games, see the recently comprehensive reviews [20,21].

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Recently, in [2,3], the authors have proposed a new predator-prey interaction, that is, the prey exhibiting herd behavior in terms of the square root of the prey population as the following model:

$$\begin{cases} \frac{du}{dt} = u(1-u) - \frac{\sqrt{uv}}{1+\alpha\sqrt{u}}, \\ \frac{dv}{dt} = \beta v \left(-\gamma + \frac{\sqrt{u}}{1+\alpha\sqrt{u}} \right), \end{cases} \quad (1.1)$$

where $u(t)$ and $v(t)$ stand for the prey and predator densities, respectively, at time t . $\beta\gamma > 0$ is the death rate of the predator in the absence of prey, $\beta > 0$ is the conversion or consumption rate of prey to predator, $\alpha \geq 0$ is related to the search efficiency of v for u and average handling time, see [2,3] in detail. When $\alpha = 0$, the authors in [2,3] have obtained some meaningful results for system (1.1) by the Poincaré–Bendixson theorem, method of phase plane analysis and the technique of numerical calculation, respectively.

In the real world, the predator and the prey may move for many reasons, such as currents and turbulent diffusion. Thus, we should consider the spatial disperse. The spatiotemporal dynamics of the predator-prey models involving spatial diffusion have been increasingly studied by many researchers. For instance, in [26,27], we have investigated the Hopf bifurcation for the delayed model (1.1) with diffusion and spatiotemporal patterns for the model (1.1) with cross-diffusion, respectively. Moreover, Turing–Hopf bifurcation, principal eigenvalue problem, travel wave solution, Turing instability and pattern formation have been recently studied, see [9,15,17,31,32,34,38,41,43,44,46]. Here, assuming the preys and the predators are in an isolate patch, we neglect the impact of migration, including immigration and emigration, and only consider the diffusion of the spatial domain. On the other hand, in the development of mathematical models for population dynamics, we know that the functional response and mortality rate of the predator are essential. Therefore, we further consider another crucial component of predator-prey relationships, that is, mortality rate including linear mortality and nonlinear mortality. Of course, linear mortality rate is intensively used by researchers, see [8,12,26,27,33,37,45]. But nonlinear mortality rates, such as quadratic mortality [4,39,42], have also been used and can lead to richer dynamics, see [39,40,42] and so on. Besides, in 1994, Cavani and Farkas in [6] introduced a new nonlinear mortality rates: hyperbolic mortality, with which predator-prey models have been studied by some scholars, see [1,7,40]. Based on the above analysis, we propose a spatial model with herd behavior and hyperbolic mortality as follows:

$$\begin{cases} \frac{\partial u}{\partial t} - d_1 \Delta u = u(1-u) - \frac{\sqrt{uv}}{1+\alpha\sqrt{u}}, & x \in (0, \pi), t > 0, \\ \frac{\partial v}{\partial t} - d_2 \Delta v = \beta v \left(-\frac{\gamma + \delta v}{1+v} + \frac{\sqrt{u}}{1+\alpha\sqrt{u}} \right), & x \in (0, \pi), t > 0, \\ u_x(0, t) = v_x(0, t) = u_x(\pi, t) = v_x(\pi, t) = 0, & t > 0, \\ u(x, 0) = u_0(x) \geq 0, v(x, 0) = v_0(x) \geq 0, & x \in (0, \pi), \end{cases} \quad (1.2)$$

where d_1 and d_2 are the positive diffusion constants for the prey and predator, respectively. The hyperbolic mortality $\frac{\gamma + \delta v}{1+v}$ of predators in absence of prey depends on the quantity of predator, γ is the mortality at low density, and δ is the maximal mortality with the natural assumption $0 < \gamma < \delta$, see [6] in detail. Here, we must point out the fact that the predator mortality in model (1.2) is a bounded and increasing function with quantity, which is other than the ones in [8,12,26,27,33,37,45] and [4,39,42], since, in [4,8,12,26,27,33,37,39,42,45], the predator mortalities are the unbounded and increasing functions with quantity.

However, to the best of our knowledge, there are no results on the above system (1.2) such as the stability and bifurcation analysis, Turing instability, and so on. So, in this paper, we shall focus on the stability of the positive equilibrium, Turing instability, Hopf bifurcation and steady state bifurcation for system (1.2). The rest of this paper is organized as follows. In Section 2, we prove the existence and uniqueness of positive equilibrium for this model (1.2) by analytical skills. In Section 3, we analyze the stability of the positive equilibrium, Turing instability, and the existence of Hopf bifurcation and steady state bifurcation. In Section 4, the formulas for determining the direction and stability of Hopf bifurcation and the type of steady state bifurcation are derived by using the normal form theory for partial differential equations. In Section 5, we illustrate our results with numerical simulations, which support and extend the theoretical results. The paper ends with a conclusion.

2. The existence and uniqueness of positive equilibrium for system (1.2)

In this section, we shall prove the existence and uniqueness of positive equilibrium for system (1.2) by analytical skills.

The equilibrium of system (1.2) satisfies the following system:

$$\begin{cases} u(1-u) - \frac{\sqrt{uv}}{1+\alpha\sqrt{u}} = 0, \\ v \left(-\frac{\gamma + \delta v}{1+v} + \frac{\sqrt{u}}{1+\alpha\sqrt{u}} \right) = 0. \end{cases} \quad (2.1)$$

Obviously, system (1.2) have two boundary equilibria $(0, 0)$ and $(1, 0)$. Now we consider the existence of positive equilibrium. Then we have the following result.

Theorem 2.1. Assume that $0 < \gamma < \delta < 1$ and $\frac{1}{\delta} - 1 < \alpha < \min\{\frac{1}{\delta} - \frac{1}{3}, \frac{1}{\gamma} - 1\}$ hold. Then (1.2) has a unique positive equilibrium (u^*, v^*) .

Proof. Define

$$r(u) = \sqrt{u}(1-u)(1+\alpha\sqrt{u}), \quad s(u) = \frac{\sqrt{u} - \gamma(1+\alpha\sqrt{u})}{\delta(1+\alpha\sqrt{u}) - \sqrt{u}}.$$

Then, from Eq. (2.1), we know that the prey and predator null-cline curves are given by $v = r(u)$ and $v = s(u)$, respectively. Denote a positive equilibrium by (u^*, v^*) , then it should satisfy

$$v^* = r(u^*) = s(u^*)$$

in the first quadrant of the (u, v) -phase plane. Since $v^* > 0$, we have $0 < u^* < 1$ by $r(u)$. Thus the question to search a

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