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Global robust stability analysis of neural networks with discrete time delays

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Abstract

Global robust convergence properties of continuous-time neural networks with discrete delays are studied. By using a Lyapunov functional, we derive a delay independent stability condition for the existence uniqueness and global robust asymptotic stability of the equilibrium point. The condition is in terms of the network parameters only and can be easily verified. It is also shown that the obtained result improves and generalizes a previously published result.

1. Introduction

In recent years, neural networks have been applied to various signal processing problems such as optimization, image processing and associative memory design. In such applications, it is important to know the convergence properties of the designed neural network. When a neural network is employed as an associative memory, it is desired that the neural network must have many equilibrium point. However, in applications to parallel computation, neural control, optimization and signal processing, the neural network should have unique equilibrium point which is globally asymptotically stable. One may refer to [1–5] and the references therein for various stability results for different neural network models. In hardware implementation of neural networks, a time delay occurs during the processing and transmission of the signals. In this case, some delay parameters are introduced into the system equations that govern the dynamical behavior of neural networks. Such neural networks have also been used in image processing and pattern classification applications. Some results concerning the equilibrium and stability properties of different classes of neural networks with delay have been reported in [6–17]. On the other hand, in hardware implementation of neural networks, the network parameters of neural system may subject to some changes due to the tolerances of electronic components employed in the design. In such cases, it is desired that the stability properties of neural network should not be affected by the small deviations in the values of the parameters. In other words, the neural network must be globally robust stable. Recently, some results concerning the global robust exponential stability of delayed neural networks have been reported [18–22]. Specially, the authors of [20] present a new linear matrix inequality and employ a Lyapunov functional to derive an important global robust stability for neural networks with a constant time delay.

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In the present paper, we will obtain a new sufficient condition for the global robust asymptotic stability of neural networks with discrete time delays. The result is shown to improve and generalize the result given in [20].

2. Preliminaries

The dynamical behaviors of delayed neural networks are assumed to be governed by the dynamics of the following set of ordinary differential equations:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j)) + u_i, \qquad i = 1,2,\ldots,n$$

which can be written in the vector-matrix form as follows:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau)) + u \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}} \in R^n$ is state vector of neural system, $C = \mathrm{diag}(c_1, c_2, \ldots, c_n)$, $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are the connection weight and delayed connection weight matrices, respectively, $u = (u_1, u_2, \ldots, u_n)^{\mathrm{T}}$ is a constant input vector, the τ_j are the delay parameters, the f_j are the activation functions, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^{\mathrm{T}}$ and $f(x(t-\tau)) = (f_1(x_1(t-\tau_1)), (f_2(x_2(t-\tau_2)), \ldots, (f_n(x_n(t-\tau_n)))^{\mathrm{T}}$.

It will be assumed that the quantities a_{ij} and b_{ij} and c_i are intervalized as follows:

$$\begin{aligned} C_I &:= \{C = \mathrm{diag}(c_i): \ 0 < \underline{C} \leqslant C \leqslant \bar{C}, \ \text{i.e.,} \ 0 < \underline{c}_i \leqslant c_i \leqslant \bar{c}_i, \ i = 1,2,\ldots,n\} \\ A_I &:= \{A = (a_{ij}): \ \underline{A} \leqslant A \leqslant \bar{A}, \ \text{i.e.,} \ \underline{a}_{ij} \leqslant a_{ij} \leqslant \bar{a}_{ij}, \ i,j = 1,2,\ldots,n\} \\ B_I &:= \{B = (b_{ij}): \ \underline{B} \leqslant B \leqslant \bar{B}, \ \text{i.e.,} \ \underline{b}_{ij} \leqslant b_{ij} \leqslant \bar{b}_{ij}, \ i,j = 1,2,\ldots,n\} \end{aligned} \tag{2}$$

In the analysis of the equilibrium and stability properties of neural networks, it has been customary to impose some restrictions on the networks parameters of the neural networks and on the activation functions. In this paper, we will assume that the activation functions $f_i(\cdot)$ are Lipschitz continuous and monotonically non-decreasing. From a mathematical point of view, this class of activation functions satisfy the following condition:

$$0 \leqslant \frac{f_i(x) - f_i(y)}{x - y} \leqslant \mu_i, \quad i = 1,2,\ldots,n, \quad \forall\, x,y \in R \tag{3}$$

where $\mu_i > 0$ denotes a Lipschitz constant.

In order to derive sufficient conditions for the global robust exponential stability of the equilibria of (1), we will need the following:

Lemma 1 [20]. *For $A \in [\underline{A}, \bar{A}]$ and $B \in [\underline{B}, \bar{B}]$, the following inequalities hold*:

$$\|A\|_2 \leqslant \|A^*\|_2 + \|A_*\|_2 \quad \textit{and} \quad \|B\|_2 \leqslant \|B^*\|_2 + \|B_*\|_2$$

where $A^* = \frac{1}{2}(\bar{A} + \underline{A}), A_* = \frac{1}{2}(\bar{A} - \underline{A}),\ B^* = \frac{1}{2}(\bar{B} + \underline{B}), B_* = \frac{1}{2}(\bar{B} - \underline{B})$.

Definition 1 [20]. The neural network defined by (1) with the parameter ranges defined by (2) is globally asymptotically robust stable if the unique equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^{\mathrm{T}}$ of the system is globally asymptotically stable for all $C \in C_I$, $A \in A_I$ and $B \in B_I$.

Definition 2 [1]. A mapping $H: R^n \to R^n$ is homeomorphism of R^n onto itself if $H \in C^0$, H is one-to-one and the inverse mapping $H^{-1} \in C^0$.

Lemma 2 [1]. *If $H(x) \in C^0$ satisfies the following conditions*:

(i) *$H(x) \neq H(y)$ for all $x \neq y$,*
(ii) *$\|H(x)\| \to \infty$ as $\|x\| \to \infty$,*

then, $H(x)$ is homeomorphism of R^n.

Define the following mapping associated with (1):

$$H(x) = -Cx + Af(x) + Bf(x) + u \tag{4}$$