Available online at www.sciencedirect.com

SCIENCE DIRECT®

Chaos, Solitons and Fractals 26 (2005) 231–246

CHAOS
SOLITONS & FRACTALS

www.elsevier.com/locate/chaos

Elliptic equation rational expansion method and new exact travelling solutions for Whitham–Broer–Kaup equations

Yong Chen [a,b,c,*], Qi Wang [c,d], Biao Li [c,d]

[a] *Department of Mathematics, Ningbo University, Ningbo 315211, China*
[b] *Department of Physics, Shanghai Jiao Tong University, Shanghai 200030, China*
[c] *Department of Applied Mathematics, Dalian University of Technology, Dalian 116024, China*
[d] *M.M. Key Lab, Chinese Academy of Sciences, Beijing 100080, China*

Accepted 9 December 2004

Communicated by Prof. M. Wadati

Abstract

Based on a new general ansatz and a general subepuation, a new general algebraic method named elliptic equation rational expansion method is devised for constructing multiple travelling wave solutions in terms of rational special function for nonlinear evolution equations (NEEs). We apply the proposed method to solve Whitham–Broer–Kaup equation and explicitly construct a series of exact solutions which include rational form solitary wave solution, rational form triangular periodic wave solutions and rational wave solutions as special cases. In addition, the links among our proposed method with the method by Fan [Chaos, Solitons & Fractals 2004;20:609], are also clarified generally.

1. Introduction

With the development of soliton theory, there have been a great amount of activities aiming to find methods for exact solutions of NEEs, such as Bäcklund transformation, Darboux transformation, Cole-Hopf transformation, various tanh methods, various Jacobi elliptic function methods, variable separation approach, Painlevé method, homogeneous balance method, similarity reduction method and so on [1–17]. Among those, the tanh method provides a straightforward and effective algorithm to obtain particular travelling solutions for a large number of NEEs. Recently, much research work has been concentrated on the various extensions and applications of the tanh method[7–17]. Recently, in Refs. [18], Fan developed a new algebraic method with symbolic computation for obtaining the above-mentioned various travelling wave solutions in a unified way, but also easily provides us with new and more general travelling wave solutions in terms of special functions such as hyperbolic, rational, triangular, Weierstrass and Jacobi elliptic double periodic functions. In Refs. [19], we extended the Fan's method [18] to a generalized method. As a result,

* Corresponding author. Address: Department of Mathematics, Ningbo University, Ningbo 315211, China. Tel.: +86 5748 7605772; fax: +86 5748 7600874.

E-mail address: chenyong@dlut.edu.cn (Y. Chen).

doi:10.1016/j.chaos.2004.12.020

we can not only successfully recover the previously known travelling wave solutions found by Fan's method but also obtain some new formal solutions. More recently, we present the Jacobi elliptic function rational expansion method [20] and the Riccati equation rational expansion method [21], in which the ansatzes are firstly express as rational form.

The present work is motivated by the desire to present a new subequation method, named elliptic equation rational expansion method, by proposing a more general ansatz so that it can be used to obtained more types and general formal solutions which contain not only the results obtained by using the various methods [6–21] but also other types of solutions. The appeal and success of the method lies in the fact: one circumvents integration to get explicit solutions based on the fact that soliton solutions are essentially of a localized nature. Writing the soliton solutions of a NEEs as the polynomials of auxiliary variables of the elliptic equation [22–24], the NEEs can changed into a nonlinear system of algebraic equations. The system can be solved with the help of symbolic computation. For illustration, we apply the generalized method to solve the Whitham–Broer–Kaup equation and successfully construct new and more general solutions including rational form solitary wave solutions, rational form triangular periodic solutions and ration wave solutions for the Whitham–Broer–Kaup equation.

This paper is organized as follows. In Section 2, we summarize the elliptic equation rational expansion method. In Section 3, we apply the method to the Whitham–Broer–Kaup equation and bring out many solutions. Conclusions will be presented in finally.

2. Summary of the elliptic equation rational expansion method

In the following we would like to outline the main steps of our method:

Step 1. For a given NEE system with some physical fields $u_i(x,y,t)$ in three variables x, y, t,

$$F_i(u_i, u_{it}, u_{ix}, u_{iy}, u_{itt}, u_{ixt}, u_{iyt}, u_{ixx}, u_{iyy}, u_{ixy}, \ldots) = 0, \tag{2.1}$$

by using the wave transformation

$$u_i(x,y,t) = U_i(\xi), \quad \xi = k(x + ly + \lambda t), \tag{2.2}$$

where k, l and λ are constants to be determined later. Then the nonlinear partial differential Eq. (2.1) is reduced to a nonlinear ordinary differential equation(ODE):

$$G_i(U_i, U_i', U_i'', \ldots) = 0. \tag{2.3}$$

Step 2. We introduce a new ansätz in terms of finite rational formal expansion in the following forms:

$$U_i(\xi) = a_{i0} + \sum_{j=1}^{m_i} \frac{a_{ij}\phi^j(\xi) + b_{ij}\phi^{j-1}(\xi)\phi'(\xi)}{(\mu\phi(\xi)+1)^j} \tag{2.4}$$

and the new variable $\phi = \phi(\xi)$ satisfying the elliptic equation [22–24]

$$\phi'^2 = \left(\frac{\mathrm{d}\phi}{\mathrm{d}\xi}\right)^2 = h_0 + h_1\phi + h_2\phi^2 + h_3\phi^3 + h_4\phi^4, \tag{2.5}$$

where h_ρ, a_{i0}, a_{ij} and b_{ij} ($\rho = 0,1,\ldots,4$; $i = 1,2,\ldots$; $j = 1,2,\ldots,m_i$) are constants to be determined later.

Step 3. The underlying mechanism for a series of fundamental solutions such as polynomial, exponential, solitary wave, rational, triangular periodic, Jacobi and Weierstrass doubly periodic solutions to occur is that differ effects that act to change wave forms in many nonlinear equations, i.e. dispersion, dissipation and nonlinearity, either separately or various combination are able to balance out. We define the degree of $U_i(\xi)$ as $D[U_i(\xi)] = n_i$, which gives rise to the degrees of other expressions as

$$D[U_i^{(\alpha)}] = n_i + \alpha, \quad D[U_i^\beta(U_j^{(\alpha)})^s] = n_i\beta + (\alpha + n_j)s. \tag{2.6}$$

Therefore we can get the value of m_i in Eq. (2.4). If n_i is a nonnegative integer, then we first make the transformation $U_i = V_i^{n_i}$.

Step 4. Substitute Eq. (2.4) into Eq. (2.3) along with Eq. (2.5) and then set all coefficients of $\phi^p(\xi)\left(\sqrt{\sum_{\rho=0}^4 h_\rho\phi^\rho}\right)^q$ ($p = 1,2,\ldots,q = 0,1.$) of the resulting system's numerator to be zero to get an over-determined system of nonlinear algebraic equations with respect to k, μ, a_{i0}, a_{ij} and b_{ij} ($i = 1,2,\ldots$; $j = 1,2,\ldots,m_i$).

Step 5. Solving the over-determined system of nonlinear algebraic equations by use of *Maple*, we would end up with the explicit expressions for k, μ, a_{i0}, a_{ij} and b_{ij} ($i = 1,2,\ldots$; $j = 1,2,\ldots,m_i$).