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El Naschie's Cantorian frames, gravitation and quantum mechanics

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Abstract

A frame is characterized by a hermitic operator with independent of direction eigenvalues. For a particular type of frames (El Naschie's frame), the hermitic operator eigenvalues may be put in correspondence with a Cantor set. This is the situation when the used coordinates, could be some values directly measurable for instance, via the mass spectrum of high energy particle physics. By using the Cayleyene metric associated to the hermitic operator and to Ernst's equations, the generalized hyperbolic stationary metrics is obtained. When imposing on the metrics the conform Minkowskian character, we obtain both the inertial nature of the Universe expansion and, by the gravitational coupling constant, the correspondence with $\varepsilon^{(\infty)}$ space–time. The quantum mechanics in the form of a Schrödinger type equation with complex eigenvalues appears in the process of frames synchronization.

1. Introduction

In quantum mechanics there exists a transcendence of directions induced by Heisenberg relations [1,2]. Indeed, the experimental apparatus must satisfy the classical laws of physics, even in the case of the quantum measurements [3]. The difference between classical and quantum laws, with respect to measure scale, arises for length, time, mass, etc. situation implied by the structure of space–time, but not for direction in space. A fixed direction at macroscopic level has physical significance in micro-physics too. Therefore, when looking for a connection between the classical and quantum theory of gravitation, we must use frames defined only by directions.

The physical frame cannot be rotated arbitrarily without a change of the phenomena described with respect to its directions. Therefore, such a frame must be chosen just as it appears in a given experiment, without the possibility of an arbitrary rotation of its directions.

On the other hand, the space–time geometry is not pre-established. It should naturally follow from the frame physical structure itself. That is why, the fractal geometry [3–6], and the fractal-Cantorian geometry [3] in particular, is a geometry of Nature. The fractal geometry has some characteristics, for example the fractal-Cantorian geometry is a compromise between the discrete and the continuum [7]. It is not simply discrete. It is transfinite discrete and has the cardinality of the continuum although it is not continuous. Seen from a far looks as if it were continuous. The result was startling because it is possible to simulate four-dimensionality using infinitely many weighted Cantor sets. This created a geometry and topology for space–time that is similar to that of radiation and obeys the same statistical distribution,

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namely a discrete gamma distribution which is known in physics as Planck distribution [8]. For details of the E-infinity analysis and the role of h in $\varepsilon^{(\infty)}$ see Ref. [9,10].

In the present paper a relationship between the frame and Cantorian space–time will be given.

2. The frame

Physics, both experimental and theoretical, is in fact based on the referential concept. The position, or more general, the motion of a body cannot be described but relative to certain referential. From the experimental point of view, the referential is given by a system of bodies or, with a certain degree of abstraction, by points "relatively fixed" one from each other, system which is the core of the measurements of distance and direction. The intervention of speculative thinking subsequently led to the detaching of the notion of coordinates of a point in a given referential: spatially speaking, the point is localized by three real numbers—its coordinates—if it is moving, also by means of time when it is localized, i.e. by a moment of time read from a clock attached to the referential.

Theoretically speaking the events or phenomena may be described by coordinates which ease up most of the computations. Usually, the system of coordinates are chosen as to satisfy a certain type of symmetry, or as to the results of the computations to be able to be report it in terms directly testable by experiments.

In principle, the computations may be performed in any coordinate system, the result being equivalent: the geometry found here has an entire arsenal of formula of transformation from one system of coordinates to another. This reciprocal transformability led subsequently to the detachment of the notion of referential from the pure experimental one mentioned above. A referential frequently used in geometry is the referential of the vectors tangent to the lines of coordinates (the so called natural referential [11]). Due to the fact that the crossings between several such referentials are easy to be expressed through derivatives of some unknown functions, the referential transformations are called "olonomous". Related to such referentials, pure mathematical, the physical one defined previously, is marked by a high degree of arbitrariness: not always its abstract defining elements—the basis vectors—may define lines of coordinates relatively to where they must be tangent. As a result, it is said that the basis of such referentials is non-olonomous [12]. However, clearly, these referentials are those used in the experimental physics, the olonomous being in general, only abstractions necessary to the calculus, as we will try to show in what follows.

Physically speaking, for a system of coordinates to correspond to reality: it is necessary to be measurable or interpretable in terms of measurable parameters. Although, in the quasi-totality of the cases the coordinates are not measurable. For example, in astronomy, the position of a star may be reported in spherical or Cartesian coordinates: azimuth, height and distance from the star. In both cases we deal with parameters with unknown significance, or effectively measured value. For example, the Cartesian coordinates are not measurable even if we consider the space absolutely Euclidian. The manner of work in astronomy consists of a second set of coordinates, for which the radial distance, deduced from physical considerations, is affected by a large arbitrariness, quantitatively speaking. Then even qualitatively, if the space is non-Euclidian, it has not the significance which was given. In this last case, even the angles do not give the real direction anymore, but a local, apparent one. Nevertheless, they keep a certain significance—as we tried to emphasize in—which is not related to translations, but only to rotations of the moving referential or of some portions of it (which to play, eventually, the part of "theodolite"). Let us not forget this fact, since we will use it in what follows.

This, because, among all the other notions used in physics, the notion of direction is endowed apparently with a certain transcendence. For example, in quantum mechanics, the experimental device must obey with enough accuracy, to the classical laws. This refers to the fact that the difference between the classical laws (applicable to the measurement gauges) and the quantum laws (applicable to the micro-objects), is extended in what concerns the measuring scale of masses, lengths and times and, in general, over any parameters which imply displacements, not over the directions, which imply only rotations. This has here two aspects: the first one concerns the intuitive fact that a direction, no matter how it has been established, may serve as referential both for microscopic, and for macroscopic ones, and the second one, concerns the fact that the imprecision relations which complies angular variables and associated operators, need a plus of precautions in definition [13], which might indicate a certain "classicism" of the respective variables.

The indiscernability of the coordinate systems, as well as the fact that in measurements one uses the non-olonomous physical referentials indicate, clearly, the necessity of reporting the calculus to these last ones. Hence, only these are used, e.g. in the operational definition of the space–time metric element or, as Ionescu Pallas observes [14], even the models of Universe defined by ingenious operational methods, as is the kinematic model of Milne, may be reformulated with the help of the notion of metrics. For the metric theories there is a strong basis: the general relativity theory of Einstein. In our opinion, what lacks in order to overcome the state of fact in physics (or in cosmology, with a higher restraint) is the physical interpretation, i.e. a physical referential, of the coordinates which enter in metrics. In other