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Chaos, Solitons and Fractals 24 (2005) 567–578
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Periodic motions and grazing in a harmonically forced, piecewise, linear oscillator with impacts

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Accepted 14 September 2004

Abstract

In this paper, an idealized, piecewise linear system is presented to model the vibration of gear transmission systems. Periodic motions in a generalized, piecewise linear oscillator with perfectly plastic impacts are predicted analytically. The analytical predictions of periodic motion are based on the mapping structures, and the generic mappings based on the discontinuous boundaries are developed. This method for the analytical prediction of the periodic motions in non-smooth dynamic systems can give all possible periodic motions based on the adequate mapping structures. The stability and bifurcation conditions for specified periodic motions are obtained. The periodic motions and grazing motion are demonstrated. This model is applicable to prediction of periodic motion in nonlinear dynamics of gear transmission systems.

1. Introduction

Piecewise linear dynamical systems are extensively used to describe engineering vibrations, such as vibration in gear box, rotor-bearing systems and elasto-plastic structures. In 1932, Hartog and Mikina [1] investigated the piecewise linear system without damping, and the closed-form solution for periodic symmetric motion was achieved. The dynamics of gears has been of great interest for many years (e.g., [2,3]) to improve gear transmission and to reduce machinery noise. However, the early investigation focused on the mesh geometries, kinematics and strength of teeth. In the early stage, the gear transmission systems were used with low speeds. The linear system was developed for the dynamics of gears and the linear model gave a very good prediction of gear vibration. Owing to high speed requirement in gear systems, the vibration and noise become very serious problems, and the linear vibration model cannot provide the adequate prediction. Therefore, in recent decades, the piecewise linear model and impact model were developed to find the origin of the vibration and noise [4]. In 1984, Pfeiffer [5] developed an impact model to describe regular and chaotic motion in gear box (also see, [6]). The other popular models considered the piecewise linear system to describe the gear transmission system [7–9]. The nonlinear frequency responses for such a model were presented. For a better understanding of the nonlinear responses in the piecewise linear system, Wong et al. [10] used the incremental harmonic balance method to obtain the periodic motions in 1991, and Kim and Noah [11] gave the stability and bifurcation analysis (also

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0960-0779/$ - see front matter
doi:10.1016/j.chaos.2004.09.028

see, [12]). This approach did not consider the transition motion, and it is very difficulty to handle the system discontinuity.

In 1983 Shaw and Holmes [13] investigated a piecewise linear system with a single discontinuity by use of the mapping technique. In 1989, Natsiavas [14] perturbed the initial conditions to determine the periodic motion and stability for a system with a symmetric, tri-linear spring. In 1990, Li et al. [15] numerical investigated the asymmetric motion of the piecewise linear system as the stiffness becomes infinite. In 1992, Kleczka et al. [16] presented periodic solutions and bifurcations of piecewise linear oscillator motion. In 2004, Luo and Menon [17] investigated global chaotic motions for a periodically forced piecewise linear system through the mapping structures. In 1991, Nordmark [18] investigated the non-periodic motion caused by the grazing bifurcation. The normal form mapping for such grazing phenomena was developed in [19,20]. Therefore, a periodically forced, piecewise, linear system with impacts is of great interest. Due to the page limitation, this paper will not focus on the mechanism of grazing motion which will be discussed in sequel. However, the grazing phenomena in this piecewise, linear system will be presented.

In this paper, periodic motions of such a piecewise linear system with impacts will be investigated analytically through the corresponding mapping structures. The stability and bifurcation conditions of the periodic motions will be achieved. Numerical simulations of periodic and chaotic motions will be presented to verify the analytical predicted results. The grazing motion is also illustrated.

2. Mechanical model

Consider a piecewise linear oscillator with impact at two displacement boundaries in Fig. 1. In this system, there are three masses m_j ($j = 1, 2, 3$) connected with the corresponding dampers with coefficients γ_j and springs of stiffness k_j. The external forcing $Q_0 \cos \Omega t$ acts on the primary mass m_2 where Q_0 and Ω are excitation amplitude and frequency, respectively. The displacement measured from the equilibrium position is denoted by x. To develop an ideal and simple model for transmission gear systems, the following assumptions are adopted:

A1: The two second masses m_j ($j = 1, 3$) are stationary before impact with the primary mass m_2.
A2: The impact between the primary mass (m_2) and the second mass (m_j ($j = 1, 3$)) is perfectly plastic at two displacement boundaries. For impacting points, the switching time will not be varied.
A3: Once they return back to the displacement boundary, the second mass m_j ($j = 1, 3$) is separated with the primary mass m_2.

After and before impacts, the motion of the system has been changed. Therefore, three equations of motion will be used to describe the entire motion. The equations of motion for this system are:

$$\ddot{x} + 2d_j\dot{x} + c_jx = A_j \cos \Omega t - b_j, \tag{1}$$

where

$$A_j = \frac{Q_0}{m_2 + m_j(1 - \delta_j^2)}, \quad c_j = \frac{k_2 + k_j(1 - \delta_j^2)}{m_2 + m_j(1 - \delta_j^2)}, \quad d_j = \frac{1}{2}\frac{\gamma_2 + \gamma_j(1 - \delta_j^2)}{m_2 + m_j(1 - \delta_j^2)}, \quad b_j = \frac{(2 - j)k_jE_j}{m_2 + m_j(1 - \delta_j^2)}. \tag{2}$$

The three sub-domains in the phase space for the discontinuous system in Eq. (1) are defined as

$$\Omega = \bigcup\nolimits_{j=1}^{3} \Omega_j, \tag{3}$$

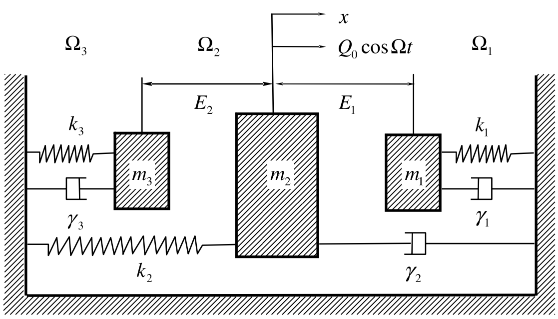

Fig. 1. Mechanical model.